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A Tree growth projection system for Northern California coniferous forests

by

Lee C. Wensel and James R. Koehler

Abstract

The individual tree growth equations used in the CAlifornia Conifer Timber Output Simulator (CACTOS) and estimates of the required coefficients are presented. Estimates are based upon measurements from a stem analysis project that also resulted in new site index and tree taper equations for northern California.

The estimates for both height and diameter growth are developed by estimating two growth components: potential and competition. Potential height growth is developed from site index curves produced from the same data and potential diameter growth is developed iteratively using a mathematical model similar to the site index model. Competition measures are based primarily upon projected crown density on the plot relative to the size of the individual tree.

Tests of model accuracy have been limited to test on the data used to fit the functions. Tests with independent data sets are possible now that the CACTOS system is being released. Such test may suggest changes in the models reported.

Mortality and ingrowth estimates were not derived as part of this study. Rather, a mortality function for Douglas-fir on California's north coast is used and ingrowth is entered directly by the user.

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The objective of this paper is to present the tree growth equations and coefficients developed for use in version 2.0 of CACTOS, the CAlifornia Conifer Timber Output Simulator (**Wensel and Daugherty, 1984**). This computerized growth and yield simulator was developed to study the effect of using alternative silvicultural systems on existing conifer forest stands in northern California. An individual tree distance-independent modelling approach is used to provide the flexibility required to enable the model to be used with forest inventory data, without the added expense of measuring tree coordinates. These equations can be used to predict the growth of trees in even-aged single or mixed species stands, multiple-aged stands (individual ages usually unknown), and also trees that are now or have been previously supressed.

The basic field data required by CACTOS for predictions are 50-year (breast height age) site index (**Biging and Wensel 1984**) for each species in the stand and the following items for each tree:

sp	species
DBH	diameter at breast height
H	total height
HCB	height to the crown base
TPA	tree weight per acre

Therefore expressions to be used in predicting growth rates were restricted to functions of the these variables. A complete list of symbols used is given in **Table 1**.

Modelling approach

Actual tree diameter and height growth are modelled as a product of the tree's potential growth and a measure of competition which restricts that tree's ability to reach its potential (**Baule,1917**). That is,

growth = (potential growth) x (competition).

Potential growth is theoretically based upon the difference of two points on the "S-shaped" growth curves. Since mathematical expressions of competition are difficult to derive theoretically, these relationships are developed empirically.

Following the methods developed by **Krumland and Wensel (1981)**, and **Krumland (1982)** competition measures are modelled as functions of stand crown density and the individual tree's competitive postion within that crown density. These measures are derived from tree DBH, total height, and live crown ratio (**Van Deusen and Biging, 1984**).

Table 1. Alphabetical list of symbols used.

A	breast-height tree age
BA	basal area
C_D	diameter growth competition
C_H	height growth competition
CA(h)	canopy cross sectional area at height (h)
CC_{66}	crown closure at 66% of tree height
D	diameter at breast height (DBH)
DBH	diameter at breast height
H	total height
H_b	total height - 4.5
HCB	height to the crown base
LCR	live crown ratio
n	number of trees used in regression
PBA_i	percent of plot basal area in species i
P_D	potential change in DBH^2
P_H	potential change in total height
S	50-year breast height age site index
S'	$S + \partial$
TPA	tree weight per acre
ΔDBH^2	change in DBH^2
ΔH	change in total height
ΔHCB	change in height to crown base
∂	site index adjustment

Possible variations of these models using additional plot data (elevation, aspect, depth of soil, etc.) may prove profitable and will be investigated in the future. However, the addition of more variables at the modelling stage means that more information will have to be specified at the time the models are used. Since even our "parsimonious" approach requires data frequently not found in forest inventories (i.e., tree height and height-to-the-crown-base on all trees), the use of additional predictors in the models will have to be approached with caution.

The height growth model given above differs from that used in the PROGNOSIS model (**Stage 1973**), a growth model in wide use by the U. S. Forest Service. PROGNOSIS uses a height growth model (**Stage 1975**) that predicts ΔH as a (logarithmic-linear) function of D, H, and ΔD. However, in the present case, D and ΔD are more sensitive to varying levels of stand density than is ΔH, which makes diameter a poor predictor of height. Also, for cutover stands, these variables alone do not reflect the competitive position of a tree within the stand after thinning. Thus, on both theoretical and practical grounds, this approach to individual tree growth modelling cannot be used here to estimate height growth.

The growth modelling approach used in STEM (**Belcher, Holdaway and Brand, 1982**) is also driven by DBH and changes in DBH and, like PROGNOSIS, is not useful in the current context.

Data

The growth coefficients presented are based upon the stem analysis data base collected in cooperation with 12 forest industry contributors for the Northern California Forest Yield Cooperative. This data set, described by **Biging (1984)**, consists of detailed stem analysis of selected trees of 6 conifer species in young-growth stands (**Table 2**). The trees were selected on 31 clusters of 3 one-fifth acre (0.08 ha) and 8 clusters of 2 one-tenth acre (0.04 ha) plots. The distribution of these plots in northern California is shown on **Figure 1** and the region, timber type and basal area for each cluster are listed in **Table 3**.

For the felled trees, diameter growth was obtained by computer analysis of digitized tree cross sections (**Biging and Wensel 1983**) and height growth was estimated by interpolation on 5, 10, and 15-year tip cuts. Further, diameter growth was estimated on the non-felled trees using increment cores. Thus, more trees were analyzed for diameter growth than for height growth.

The individual trees were "de-grown" by subtracting the 5-year diameter and height growth from current measurements. The de-growing process allowed us to model growth rates as a function of the tree size at the beginning of the previous growth cycle. However, the live crown ratio was assumed to remain the same for the growth cycle since we had no measure of the change in live crown ratio. Further explanation of the de-growing process can be found in **Appendix A**.

Table 2. Definition of species codes used

code	definition
PP	ponderosa pine *Pinus ponderosa* (Laws.)
SP	sugar pine *Pinus lambertiana* (Dougl.)
DF	Douglas-fir *Pseudotsuga menziesii* (Mirb.) Franco
WF	white fir *Abies concolor* (Gord. and Glend.) Lindl.
RF	red fir *Abies magnifica* (A. Murr.)
IC	incense cedar *Libocedrus decurrens* (Torr.)

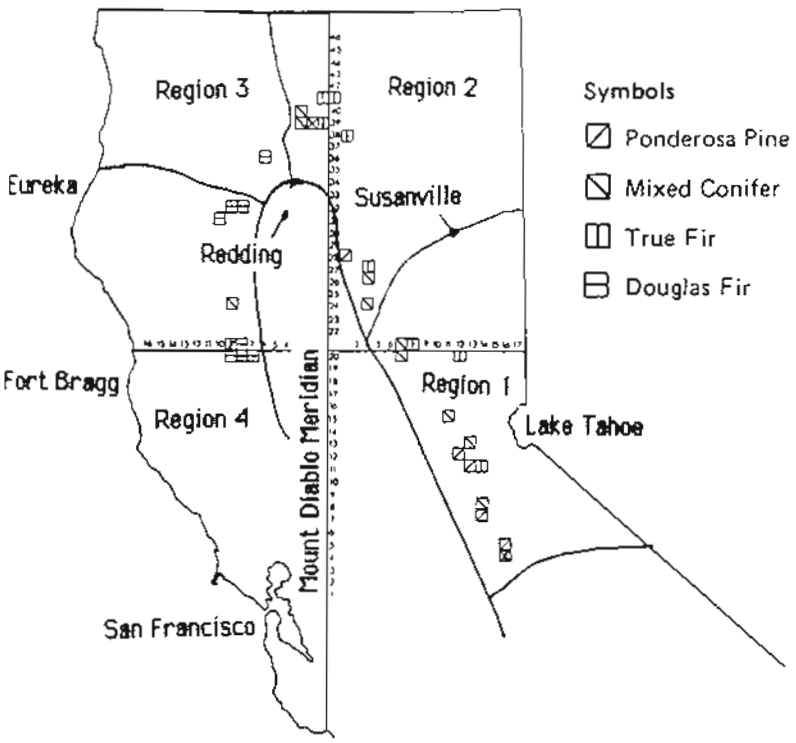

Figure 1. Distribution of stem analysis plots in Northern California.

Table 3. Stand statistics by clusters for stem analysis data base.

CLUSTER	CLUSTER[1/] CODE	REGION	TIMBER TYPE	BASAL AREA (>6)
1	A	1	TF	210
2	B	1	MC	203
3	C	1	MC	197
4	D	1	MC	186
21	E	2	TF	76
22	F	2	TF	112
23	G	2	MC	138
24	H	3	DF	70
25	I	3	DF	197
28	J	2	PP	200
29	K	2	MC	176
41	L	2	MC	278
42	M	2	MC	245
43	N	2	TF	250
44	O	2	PP	233
51	P	2	TF	252
52	Q	2	TF	249
71	R	1	MC	195
72	S	1	MC	226
73	T	1	TF	208
81	U	1	MC	282
82	V	1	MC	179
84	W	1	TF	258
85	X	1	MC	305
91	Y	1	MC	176
92	Z	1	MC	227
93	0	1	PP	222
101	1	2	TF	191
102	2	1	TF	264
103	3	3	DF	142
104	4	3	MC	101
111	5	4	MC	145
112	6	4	MC	147
113	7	4	MC	184
114	8	4	MC	178
115	9	4	DF	139
116	#	4	MC	171
117	$	4	MC	280
118	%	4	MC	208

[1/] Cluster codes are used in residual plots found in **Appendix C**.

Height growth

The height growth model form used is

$$\Delta H = (\text{potential change in height}) \times (\text{height competition})$$

or

$$\Delta H = (P_H)(C_H)$$

where ΔH is the change in height for 5 years, P_H is the potential height growth, and C_H is the competition that keeps the tree from reaching its potential height growth rate.

The potential height growth is derived from the unconstrained site index curves developed by **Biging and Wensel (1984)**:

$$H = 4.5 + b_0 S^{b_1} [\, 1 - \exp(-b_2 A) \,]^{b_3} \qquad [1]$$

where H is the total tree height at breast-height age (A)for site index (S) and $b_0 = 2.93243$, $b_1 = 0.89$, $b_2 = 0.024$, and $b_3 = 1.81845$. As proposed by Biging and Wensel, the same site index curve is used for all species, but each species on a site is likely to have a different site index.

The expression for height growth is based on the proposition that the potential change in height follows the site index curve from the tree's current height at age A (unknown) to what it would be at age (A + 5). Thus, using an inverted form of equation **[1]**, an "artificial" tree age is computed for a tree of height H on site index S. Adding 5 years to this age and substituting into equation **[1]**, and subtracting current height, yields the expression for the height growth for the 5 year cycle (see **Appendix B**). This process is illustrated in Figure 2.

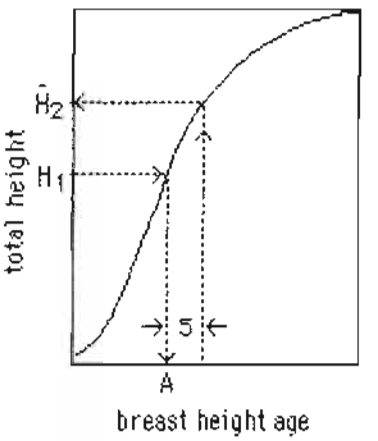

Figure 2. Illustration of height growth potential determination.

Explicitly,

$$P_H = 4.5 + [c_0 S^{c_1} + c_2 H_b^{c_3}]^{1/c_3} - H \qquad [2]$$

where

$$c_0 = [1 - \exp(-5\, b_2)]\, b_0^{(1/b_3)}$$
$$c_1 = b_1 / b_3$$
$$c_2 = \exp(-5\, b_2)$$
$$c_3 = 1 / b_3$$
$$H_b = H - 4.5$$

Using the values of b_i provided above produces $c_0 = 0.204$, $c_1 = 0.489$, $c_2 = 0.887$, and $c_3 = 0.550$.

Note that site index reflects the average height of dominant and codominant trees and, therefore, the potential height growth could be somewhat higher than that obtained by using equation **[2]**. As a result, while fitting the competition coefficients (below), a site index adjustment (∂) was computed for each species such that the potential height growth is computed using site index S'; or

$$S' = S + \partial$$

in place of S in equation **[2]**. The adjustments are given in **Table 4**.

The form of the height growth competition (C_H) factor is as follows:

$$C_H = \{ d_1 / [1 + \exp(4 - d_2\, LCR)] \} \times \exp(-d_3\, CC_{66}^{\,d_4}\, PBA^{d_5}) \qquad [3]$$

where LCR is the live crown ratio, CC_{66} is the crown area at 66% of the subject tree's height, PBA is the proportion of the basal area of that species in the stand and the coefficients $d_i (i=1,\ldots,5)$, estimated for each species using nonlinear regression, are given in **Table 4** along with overall fitting statistics.

Since CC_{66} is not easily measureable on field plots, it is computed using the crown models developed on the felled trees of this study **(Van Deusen and Biging, 1984)**. LCR is used here to reflect the ability of a tree to respond to changes in stand density while CC_{66} is used to reflect the density of the plot as it affects the photosynthetic portion of the crown **(Krumland and Wensel 1981)**. The computation of CC_{66} is discussed further below.

Note that the basal area of the species (PBA) is only used for ponderosa pine (other species have a zero power). This provides a basis for expressing the effect of root competition in ponderosa pine stands. Also, some of the coefficients (marked with an asterisk in **Table 4**) were held constant for the regressions. This is because the sample data were inadequate to fit all coefficients for those species. For example, since the red fir and incense cedar samples were small, all of the coefficients except the scale coefficient, d_1, were set equal to those obtained for white fir. This produced more

stable results than fitting all coefficients to the data. Also, site index for incense cedar was set to 70 because no site index measurements were taken[1/].

The fit statistics for the final models for each species appear in **Table 4**, where the standard errors range from 1.20 to 1.93 feet.

Table 4. Coefficients (and standard errors) for height growth competition, C_H, equation [3], and site index adjustements a.

Coef.	PP	SP	DF	WF	RF	IC
a	31.6 (6.8)	20.2 (5.7)	22.3 (7.0)	5.8 (2.6)	5.8*	5.8*
d_1	1.00*	1.00*	1.00*	1.00*	0.6488 (0.047)	0.7324 (0.037)
d_2	31.2*	22.4 (4.4)	30.3 (6.9)	31.4 (6.7)	31.4*	31.4*
d_3	1.6281 (0.170)	1.4086 (0.198)	0.9050 (0.110)	1.6951 (0.2594)	1.695*	1.695*
d_4	0.4630 (0.088)	0.7573 (0.163)	0.2932 (0.104)	1.4125 (0.235)	1.4125*	1.4125*
d_5	0.3083 (0.089)	0.0*	0.0*	0.0*	0.0*	0.0*
n	152	47	145	279	37	71
MSE	2.58	1.45	2.3	3.72	1.62	2.42

*Coefficient held constant for regression.

The residual plots (Appendix C) for ponderosa pine, Douglas-fir, and white fir show that most of the errors are within 3 feet, with standard errors (square root of the mean square error, MSE) ranging from from 1.2 to 1.93 feet. For white fir, especially, there is a distinct skewness to the residuals. However, the residuals show no bias with respect to either predicted height growth or height.

Thus, as judged against the stem analysis data used for estimation, the equations appear to fi quite well. For height growth this may be the best data set to use for verification -- at least as far as the quality of the measurements is concerned. Vagaries in other sources of height growth data

1/ This arbitrary assignment of site index for incense cedar was intended to be a temporary convenience. However, tests of the resulting growth equations by the cooperators showed excellent agreement between predicted and measured growth rates for this species.

suggest that it may be some time before a meaningful independent verification of the height growth data is possible.

Diameter growth

The model for diameter growth is of the same general form as that used for height growth. That is, the change in DBH squared, ΔDBH is given by

$$\Delta DBH^2 = (\text{ potential change in } DBH^2)(\text{diameter competition})$$

or

$$\Delta DBH^2 = (P_D)(C_D)$$

Diameter growth curves analogous to site index curves do not exist since diameter growth is more sensitive than height growth to competition, Therefore the following procedure was developed for estimating the diameter growth potential component.

To get "starting values" for the estimation process, ten dominant ponderosa pine trees with little evidence of past suppression, each of which had at least 8 growth measurements, were fitted separately to the equation:

$$DBH^2 = b_0 S^{b_1} [1 - \exp(-b_2 A)]^{b_3} \qquad [4]$$

where S is the species-specific site index, A is the breast height age of the tree and b_0, b_1, b_2, and b_3 are coefficents. Each coefficient was then averaged over the 10 trees to obtain initial estimates of the coefficients for all species.

In a manner analogous to the height growth derivation given in **Appendix B**, equation **[4]** can be converted to:

$$P_D = [c_0 S^{c_1} + c_2 D^{2c_3}]^{1/c_3} - D^2 \qquad [5]$$

where

$$c_0 = [1 - \exp(-5 b_2)] b_0^{(1/b_3)}$$
$$c_1 = b_1 / b_3$$
$$c_2 = \exp(-5 b_2)$$
$$c_3 = 1 / b_3$$

Using the estimates of the b_i's ($i=0,\ldots,3$) found from equation **[4]**, initial starting values of the c_i's ($i=0,\ldots3$) in equation **[5]** where calculated for use in nonlinear regression. An exploratory form of the competition component was held fixed during the non-linear regression of the potential component. Unstable estimates of the coefficients for sugar pine, incense cedar and red fir, and the high degree of correlation between coefficients, led to the adoption of the white fir coefficients to

express the potential DBH^2 growth for the above mentioned species due to the stability of estimation and reasonability of results for white fir. Failures, if any, of this convention to represent the actual potential diameter growth should be compensated for in the coefficients of the competition model.

In developing the expression of competition for diameter growth, it appeared that diameter growth was also a function of the amount of crown available in absolute terms. Thus in addition to crown closure (CC_{66}), the tree's total crown volume (CV) was used for diameter competition, C_D, as follows:

$$C_D = \exp (d_1 \, CC_{66}^{d_2} \, CV^{d_3}) \qquad [6]$$

where d_1, d_2, and d_3 are coefficients to be estimated and CV and CC_{66} are described in the next section.

Attempts at a simultaneous solution of all coefficents confounded the potential and competition effects. Therefore this approach was abandoned. Instead an iterative process was used to obtain coefficents of equations **[5]** and **[6]** for white fir, ponderosa pine and Douglas fir. First the potential coefficents (c_i's, i=0,...,3) were held constant while the competition (d_i's, i=0,...,3) were obtained; then new potential coefficents were obtained while the new d_i's were held constant. This iterative technique allows for more stable coefficent estimates with lower intra-component coefficent correlations. The iteration process was stopped when estimates of the coefficients changed less than 1% of their standard errors

The final coefficent estimates and standard errors can be found in **Table 5**. The standard errors (square roots of the mean square errors) vary from 9 to 12 (inches^2). Expressing these standard errors in terms of DBH growth, they vary from 0.37 to 0.49 inches for 12 inch trees, and 0.19 to 0.25 inches for 24 inch trees. The residual of the predicted DBH^2 growth are given in **Appendix D**. The positive skewness noted in the height growth residuals is even more obvious in predicting DBH^2 growth. Also, the residual variances increase with the size of the predictions as well as with the size (DBH) of the trees. However, though not shown in **Appendix D**, the relative errors decrease.

Verification of the diameter growth predictions is expected from an independent fit to the permanent plot data collected by the cooperators, as well as through separate tests made by the cooperators. This is likely to result in revisions of some, or all, of the coefficients in the DBH^2 growth equations.

Table 5. Coefficients (and standard errors) for diameter growth equations [4] and [5].

Coef.	PP	SP	DF	WF	RF	IC
			POTENTIAL #			
c_0	0.4262 (0.142)	0.1763*	0.1301 (0.044)	0.1763 (0.020)	0.1763*	0.1763*
c_1	0.1939 (0.063)	0.1490*	0.4653 (0.085)	0.1490 (0.023)	0.1490*	0.1490*
c_2	0.95*	0.9*	0.95*	0.9*	0.9*	0.9*
c_3	0.3461 (0.035)	0.1562*	0.3481 (0.038)	0.1562 (0.020)	0.1562*	0.1562*
			COMPETITION #			
d_1	-12.002 (2.164)	-8.044 (3.495)	-4.849 (1.078)	-2.544 (0.107)	-10.500 (1.702)	-6.743 (0.955)
d_2	0.4127 (0.052)	0.8290 (0.143)	0.4352 (0.48)	0.8458 (0.034)	0.1941 (0.131)	0.4876 (0.068)
d_3	-0.2241 (0.024)	-0.1835 (0.057)	-0.1255 (0.026)	0.000*	-0.28*	-0.1642 (0.023)
n	532	100	386	914	126	420
MSE	129.7	139.0	93.5	114.9	92.3	81.7

* Coefficent held constant for regression.

\# Coefficents and standard errors shown represent the results of the last potential/competition iteration.

Estimation of crown volume and closure.

The crown measurements used in the competition equations **[3]** and **[6]** were developed by **Van Deusen and Biging (1984)** using the same felled tree data used in this study (see above). The tree crown volume used in equation **[6]** is estimated by

$$CV = a\,D^b\,H^c\,LCR^d \qquad [7]$$

where, as before, D is tree DBH, H is tree total height, LCR is tree live crown ratio, and a, b, c, and d are coefficients given, by species, in **Table 6.** Further, the crown area at height h, CA(h), is given by

$$CA(h) = k\,CV\,(H - HCB)^{-1} \qquad 0 < h \le HCB \qquad [8]$$

and

$$CA(h) = k\,CV\,(H - h)^{k-1}\,(H-HCB)^{-k} \qquad HCB < h \le H \qquad [9]$$

where HCB is the height to the crown base, CV is estimated using equation **[7]** and the values of k are given by species in **Table 6**. These models, illustrated in **Figure 3**, define the projection from the base of the crown to ground level (equation [8]) as well as the crown taper from the base of the crown to the tip.

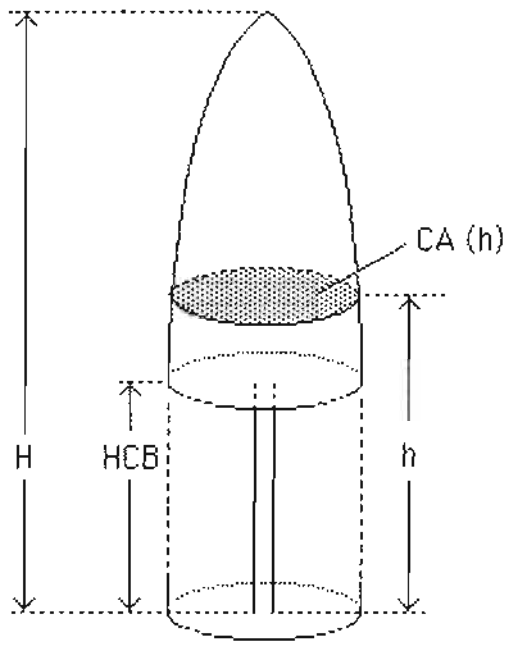

Figure 3. Model for crown cross sectional area (CA) at height h.

Using equations **[8]** and **[9]** we can compute CC_{66} for each tree in the stand as follows, for tree i, i = 1, 2, 3, ···, n:

(1) Let $h_i = 0.66\,H_i$.

(2) Use equations **[8]** and **[9]** to compute the crown area of all trees in the stand at height h_i, denoting this as $CA_j(h_i)$, for j = 1, 2, . . . , n.

(3) For tree i, express the crown area of the stand at height h_i as

$$CC66_i = \Sigma_j \, CA_j(h_i) \times TPA_j \, / \, 43{,}560 \qquad [10]$$

where TPA_j, the number of trees per acre for tree j, is used as a weight in the sum of the crown areas of all trees at height h_i. Dividing by 43,560, the number of square feet in an acre, converts the absolute area to relative area as is commonly used for expressing crown closure.

This process is illustrated in **Figure 4** where the values of CC_{66} are illustrated for the trees shaded. In (a), the center tree is shorter than the others, giving it a higher value of CC_{66} than for the left tree in (b) or the right tree in (c). Note that, as measured by CC_{66}, the center tree contributes nothing to the crown competition for the tallest tree shown in (c).

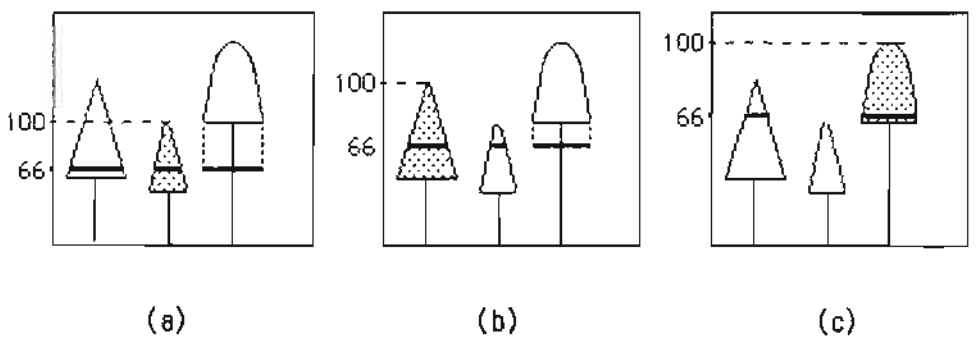

Figure 4. Crown cross sectional area at 66% of tree's height. (CC_{66} shown by bold line for shaded tree.)

Table 6. Coefficients for crown models equations [7] and [9].*

Coef.	PP	SP	DF	WF	RF	IC
a	10.284	0.250	0.233	6.045	0.175	1.081
b	1.928	1.290	0.919	1.414	2.026	1.083
c	0.424	1.622	1.897	0.787	1.203	1.465
d	2.035	1.627	1.890	1.480	2.166	1.168
k	1.868	1.976	2.063	2.152	2.042	1.938

* Fit statistics and complete information can be found in Research Note #9 (**Van Deusen and Biging, 1984**).

While the above process expresses the conceptual formation of CC_{66}, it is not efficient computationally. The procedure actually used by CACTOS to compute CC_{66} is an approximation of that given above. First, a vector of crown closure values is computed at 10-foot vertical intervals. Then, for each tree in turn, the crown closure was obtained at 66% of the tree's height by linear interpolation. This approximation reduces the number of computational steps from from n^2 to kn, where n is the number of tree records and k is the maximum tree height on the stand divided by 10.

Predicting changes in height to the crown base.

Height to the crown base was used above in developing measures of crown competition. On the first growth cycle the height to the crown base measured in the field is used. For subsequent cycles the crown base may be subject to change, depending upon the changes in density of the plot.

The change in the height to the crown base is estimated using the crown base model used in STAG (**Van Deusen and Biging, 1984**). STAG estimates the height to the crown base as follows:

$$HCB = H(1 - e^{-p}) \qquad [11]$$

where

$$p = [c_1 + c_2 \log(BA6) + c_3 (D/H)]^2 \qquad [12]$$

BA6 is the basal area of trees with DBH greater than or equal to 5.5 inches and the coefficents c_1, c_2, and c_3 are given by species in **Table 7**. Change in the height to the crown base is then estimated by taking the difference of predictions from this model at 2 different times. That is,

$$\Delta HCB = HCB(2) - HCB(1), \quad \text{if } HCB(2) - HCB(1) > 0$$
$$\Delta HCB = 0, \quad \text{elsewhere} \qquad [13]$$

Therefore Δ HCB is non-decreasing.

However, all height-to-crown-base estimates have high standard errors and this process may need refinement when additional crown data are available from remeasured plots. Further, the restriction that $\Delta HCB \geq 0$ is theoretically sound for most conifers but may have to be revised for hardwood species.

Table 7. Coefficients for height to crown base models equations [12].

Coef.	PP	SP	DF	WF	RF	IC
c_1	0.895	0.619	0.534	0.403	0.892	0.532
c_2	0.046	0.035	0.086	0.069	0.0	0.500
c_3	-1.543	0.0	-1.147	0.0	-0.458	0.0

Mortality

Estimates of tree mortality rates are necessary in order to make realistic projections. Failure to incorporate reductions in stocking levels due to mortality can produce misleading estimates of net change, especially when projections are for more that one 5-year growth cycle.

In the absence of mortality estimates from the current study, the model used in CACTOS to estimate mortality rates is the same as that used in CRYPTOS. The basic model, given by **Krumland, Dye, and Wensel (1977)** following earlier work by **Hamilton (1974)** and **Hamilton and Edwards (1976)**, expresses the annual probability of a tree dying (p) as

$$p = 1 / \{1 + \exp[-(B_0 + B_1 RBA + B_2 DBH + B_3 TPA + B_4 LCR)]\}$$

[14]

with the coefficients given in **Table 8**, and the variables defined as follows:

RBA	relative basal area (ave. tree basal area / tree basal area)
DBH	diameter at breast height
TPA	number of trees per acre
LCR	live crown ratio

Table 8. Coefficients for mortality models equations [14].

Coef.	conifers	hardwoods
B_0	-3.71	-4.621
B_1	0.0109	0.00167
B_2	-0.110	-0.1159
B_3	-0.000445	-0.00206
B_4	0.0	0.010

The probability of mortality given in **[14]** is then used to adjust the number of trees per acre (TPA') represented by the tree as follows:

$$TPA' = TPA * (1 - p)^{yf} \qquad [15]$$

where y is the number of years in the growth cycle and f is used as a "lag" variable to account for periods of growth lost after cutting,

$$f = 1 - (\text{no. of years lost}) / (\text{length of period}) \qquad [16]$$

In the current applications, y = 5 years. The lag time is set by the CACTOS user and was not investigated here. Projections for more than 5 years are made in 5-year increments.

In the absence of data sufficient to produce interim mortality estimates from the current study, the mortality coefficients used by CRYPTOS for Douglas fir, a moderately tolerant conifer, were used for all conifers. This will likely under estimate mortality for pines and over estimate mortality for the white and red firs. The mortality estimates for all hardwoods were set equal to those estimated from hardwood data on the north coast.

These interim mortality estimates will be replaced as soon as remeasurement data are available to allow better estimates to be computed.

Ingrowth

Currently ingrowth must be added by the user directly. Once ingrowth has been added into CACTOS, those new tree records are treated in the same fashion as the initial tree records. That is, no distiction is made between established tree records and new ingrowth tree records. New ingrowth models are currently being investigated.

Discussion

The models and coefficients presented above represent the principal growth estimators installed in the CACTOS program (**Wensel and Daugherty, 1984**). For each of the height, diameter, and crown growth models, the same basic model forms were used for all species but separate coefficients were estimated for each. This enabled us to use the larger data sets (usually white fir) to determine the model form and then estimate the required coefficients with greater confidence for each species.

Measures of the variability associated with the estimates of the model coefficients are given in **Table 4 and 5**. However, the standard errors of real interest are those determined by combining all of the models to produce estimates of the change in volumes, instead of only looking at the precision of the estimates of the individual coefficents.

An examination of the residuals in **Appendices C** and **D** suggest that the height growth and diameter-squared growth models fit quite well. However, these models have not yet been tested against an independent data set. When this is done more meaningful estimates of their reliability will be developed. However, before additional testing will be done, the diameter growth rates will be recomputed using the more extensive 710 permanent plot data base.

As noted above, the mortality estimates used are taken directly from the redwood model, CRYPTOS (**Krumland and Wensel, 1981**). Since they have not been derived from the stand types common to the areas studied here, they should not be considered to be accurate. Reliable estimates of mortality, and estimates of ingrowth rates and patterns, are important topics for future research.

One of the functions of this Research Note is to solicit review comments on both the estimates produced by CACTOS and the procedures outlined. Estimates provided here include both those computed as part of this study and those included merely for completeness. Future research will provide estimates of mortality and ingrowth rates as well as revised estimates of height and diameter growth.

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APPENDIX A

The "de-growing" process

Initial stocking estimates are needed to prepare the growth data for analysis. This can be accomplished by subtracting the measured past growth from the current size. Consequently, growth estimates are needed for all trees on the plots to be analyzed. While most of the felled trees have diameter and height growth measurements, only approximately half of the non-felled trees have diameter growth (from borings) and none of the non-felled trees have height growth information.

To compute the initial size of each tree on the plot, the current size minus the last 5-years growth is used. For trees which have no growth information, the following procedure was derived based upon an assumption of compound growth. That is,

$$Y_i = (1 + p)\, Y_{i-1}$$

where

Y_i is the size of tree at time i (i=0,1,2) and

p is the compound rate.

We know the current size Y_1 from our inventory, and we can predict Y_2 using an initial growth model. Hence, we can predict p as follows:

$$\hat{p} = (\hat{Y}_2 - Y_1) / Y_1$$

and therefore we can predict the initial size Y_0 by:

$$\hat{Y}_0 = Y_1 / (1 + \hat{p}) = Y_1 / (1 + (\hat{Y}_2 - Y_1)/Y_1)$$
$$= Y_1^2 / \hat{Y}_2$$

The effect of using the compound rather than the simple growth rates is minor for large trees and for all trees with small growth rates. (See **Table A.1**)

DBH (in.)	DBH increment (in.)	
	.5	1.0
10	.998	.990
20	.999	.998

Table A.1. Ratio of simple to compound growth rates in predicting prior DBH.

The effect of any errors in the assumptions used here will be negligible since the estimates are only used to "de-grow" tree in order to obtain competition measures.

APPENDIX B

Derivation of potential height growth expression

The potential height growth equation is derived from unconstrained the site index expression used by Biging and Wensel (1984),

$$H = 4.5 + b_0 S^{b_1} \{ 1 - \exp(-b_2 A) \}^{b_3} \qquad [1]$$

which expresses the tree's total height (H) as a function of its breast- height age (A) and the site index (S), where the b_i's ($i=0,\ldots,3$) are constants given above. Since the modelling is being done for uneven-aged forest stands, the tree ages will not usually be known. Solving equation [1] for tree age yields the expression

$$A = -(1/b_2) \log\{ 1 - [(H-4.5)/(b_0 S^{b_1})]^{1/b_3} \} \qquad [B.1]$$

The height of the tree at the end of a growth cycle of years can now be expressed as

$$H_2 = 4.5 + b_0 S^{b_1} \{ 1 - \exp(-b_2 (A+5)) \}^{b_3} \qquad [B.2]$$

where we substitute equation [B.1] for A and "simplify" as follows:

$$H_2 = 4.5 + b_0 S^{b_1} \{ 1 - \exp(-b_2(-1/b_2 \log\{ 1 - [(H-4.5)/(b_0 S^{b_1})]^{1/b_3} \} + 5)) \}^{b_3} \qquad [B.3]$$

$$H_2 = 4.5 + b_0 S^{b_1} \{ 1 - \exp(-5b_2) [1 - [(H-4.5)/(b_0 S^{b_1})]^{1/b_3}] \}^{b_3} \qquad [B.4]$$

$$H_2 = 4.5 + b_0 S^{b_1} \{ 1 - \exp(-5b_2) + \exp(-b_2 5) [(H-4.5)/(b_0 S^{b_1})]^{1/b_3} \}^{b_3} \qquad [B.5]$$

$$H_2 = 4.5 + \{ (1-\exp(-5b_2)) (b_0 S^{b_1})^{1/b_3} + \exp(-5b_2) (H-4.5)^{1/b_3} \}^{b_3} \qquad [B.6]$$

Finally, subtracting the current height, we have

$$P_H = H_2 - H$$

or

$$P_H = 4.5 + \{ c_0 S^{c_1} + c_2 (H-4.5)^{c_3} \}^{1/c_3} - H \qquad (2)$$

where

$$c_0 = (1 - \exp(-5b_2))\ b_0^{1/b_3}$$

$$c_1 = b_1 / b_3$$

$$c_2 = \exp(-5b_2)$$

$$c_3 = 1 / b_3$$

From Biging and Wensel (1984), $b_0 = 2.9324$, $b_1 = 0.8900$, $b_2 = 0.024$, and $b_3 = 1.81845$ yielding

$$c_0 = 0.2043$$

$$c_1 = 0.4894$$

$$c_2 = 0.8869$$

$$c_3 = 0.5499.$$

APPENDIX C

Height growth residual plots

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Figure C.3 Residuals plotted on predicted height growth--Douglas-fir.

Figure C.4 Residuals plotted on total height--Douglas-fir.*

Figure C.5 Residuals plotted on predicted height growth--white fir.

Figure C.6 Residuals plotted on total height--white fir.*

* Note that the plots of residuals verses total height were plotted using cluster codes.

Figure C.1 Residuals plotted on predicted height growth--ponderosa pine.

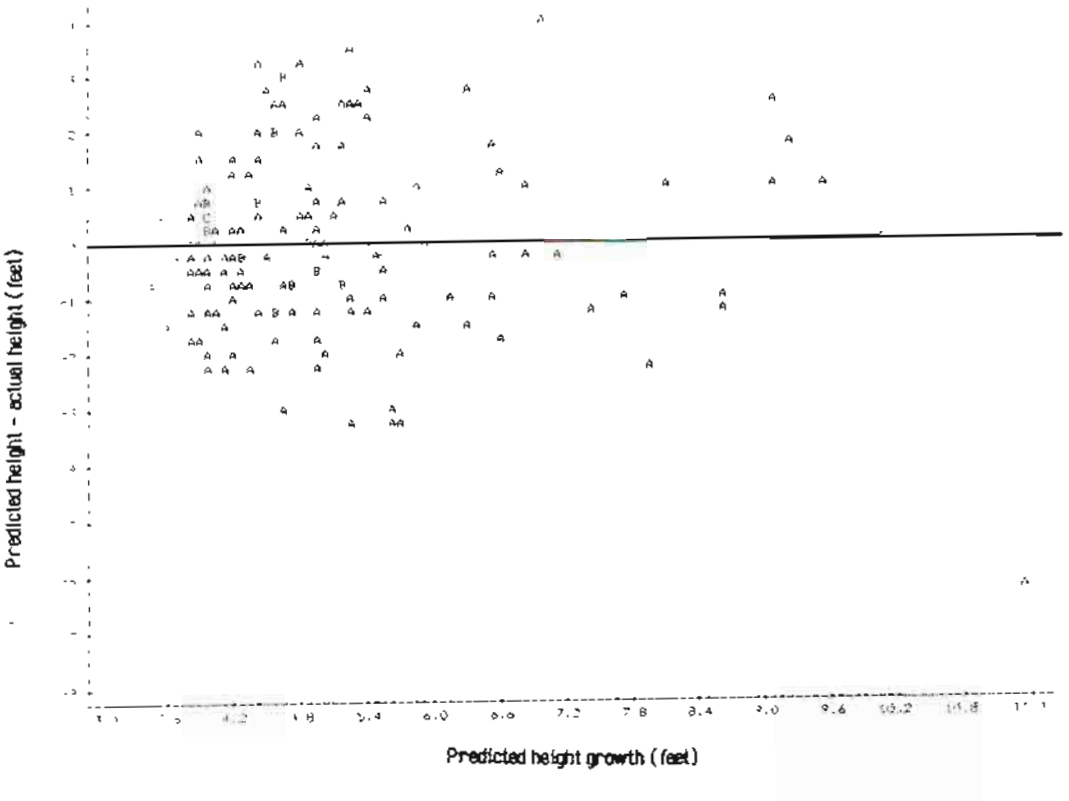


Figure C.2 Residuals plotted on total height--ponderosa pine.

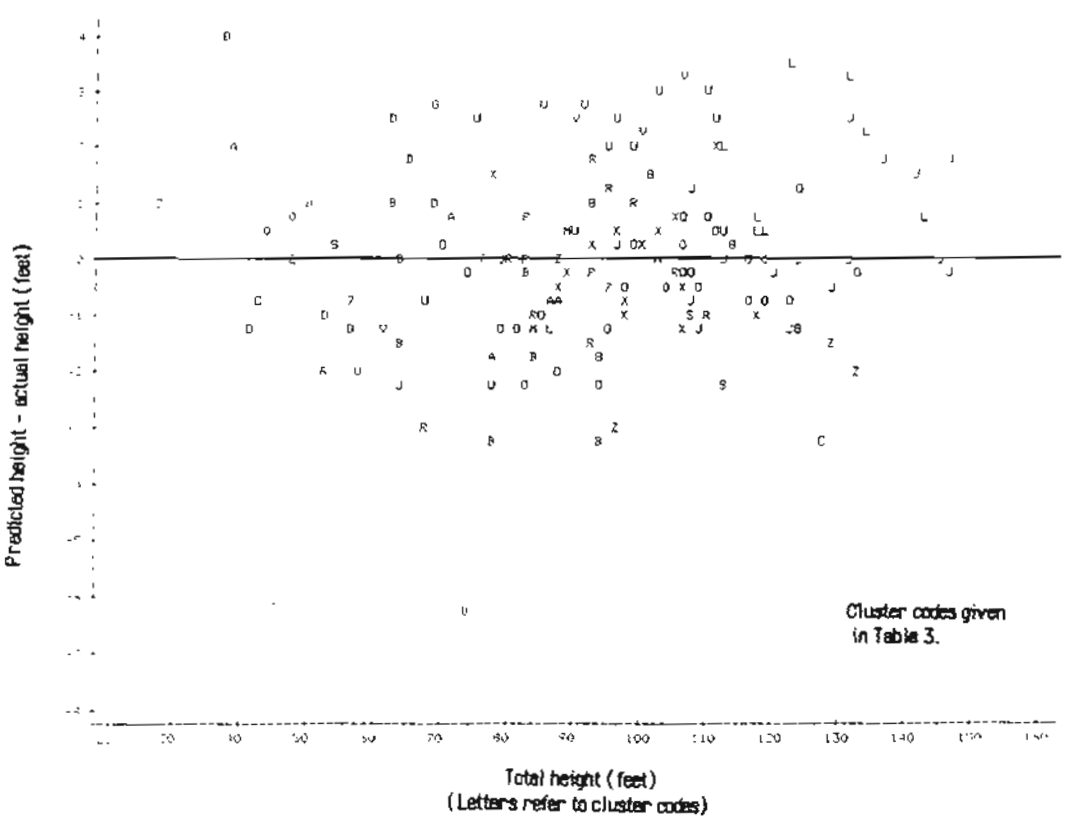


Figure C.3 Residuals plotted on predicted height growth--Douglas-fir.

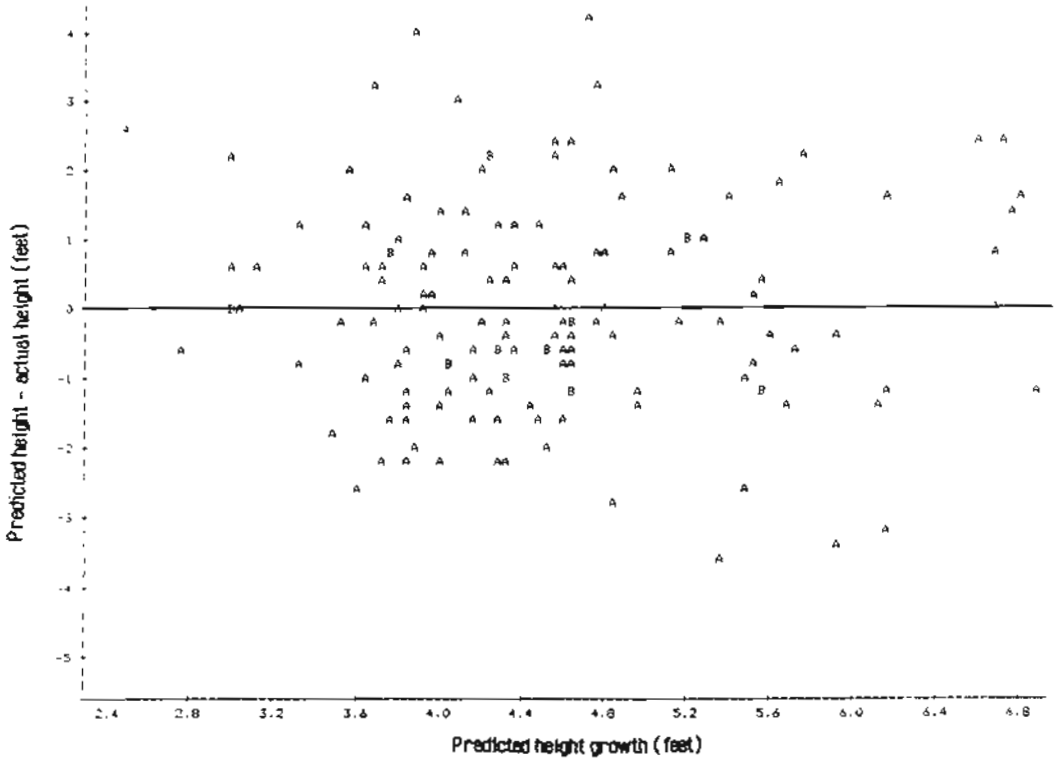


Figure C.4 Residuals plotted on total height--Douglas-fir.

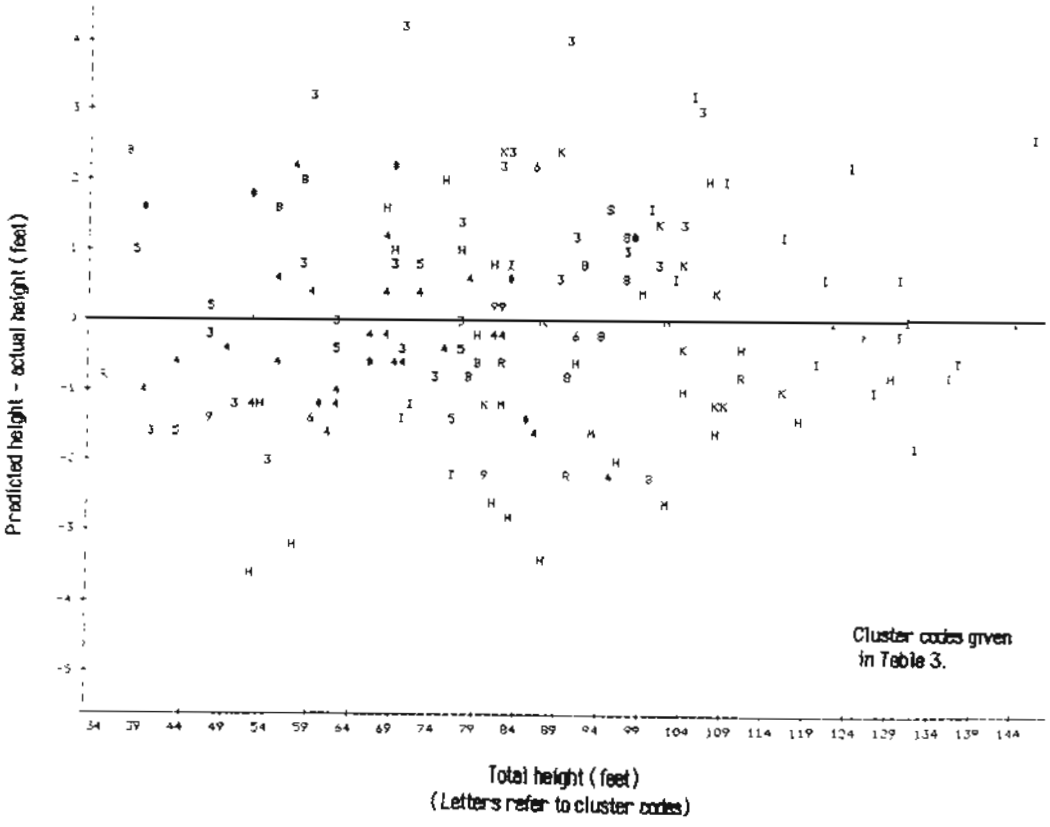


Figure C.5 Residuals plotted on predicted height growth--white fir.

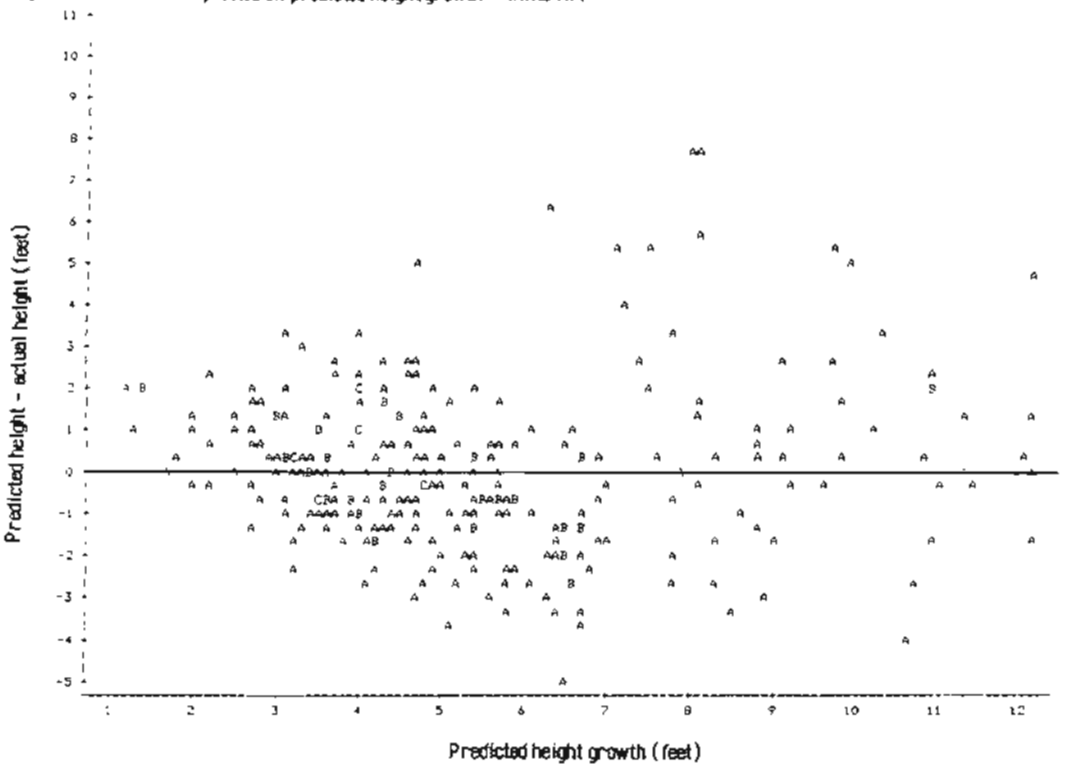
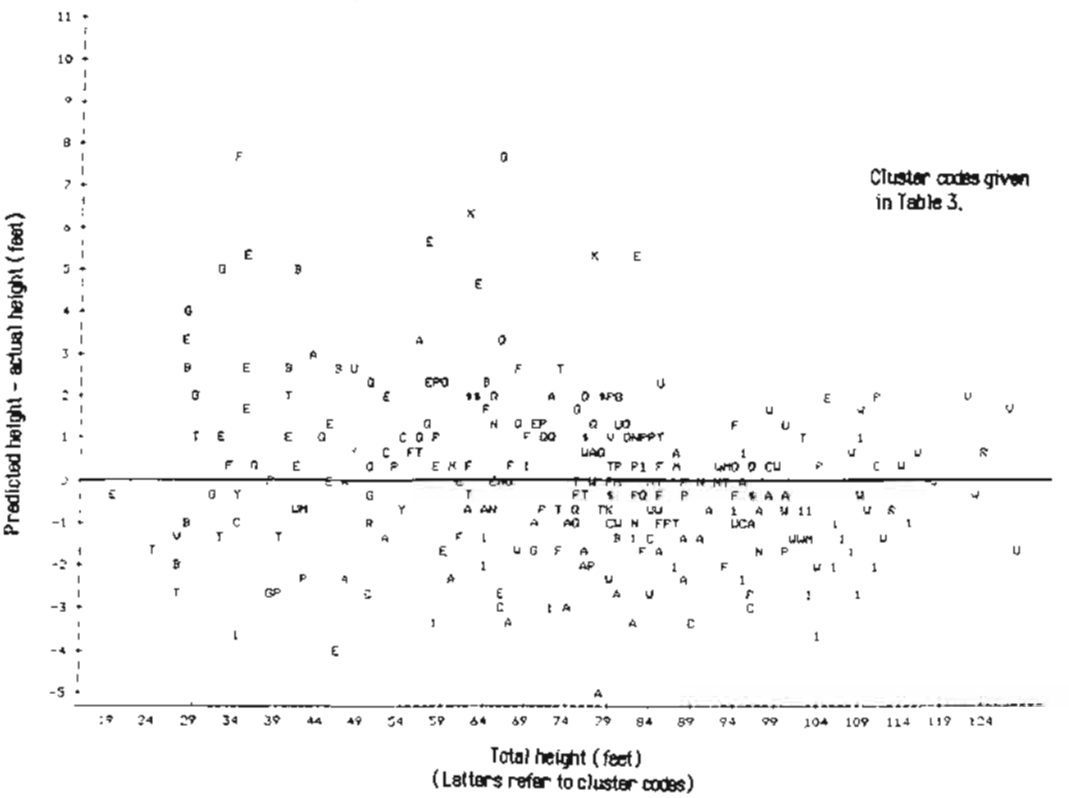


Figure C.6 Residuals plotted on total height--white fir.



APPENDIX D

Diameter-squared growth residual plots

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Figure D.4 Residuals plotted on diameter--Douglas-fir.

Figure D.5 Residuals plotted on predicted diameter-squared growth--white fir.

Figure D.6 Residuals plotted on diameter--white fir.

Figure D.1 Residuals plotted on predicted diameter-squared growth--ponderosa pine.

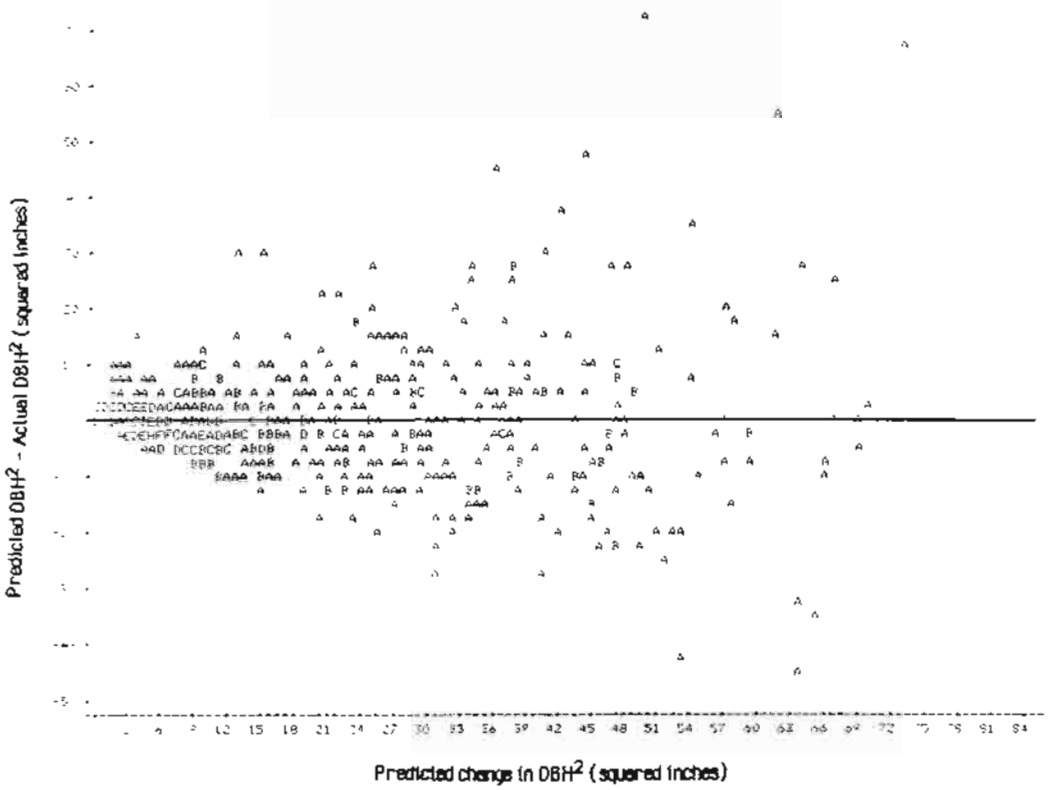

Figure D.2 Residuals plotted on diameter--ponderosa pine.

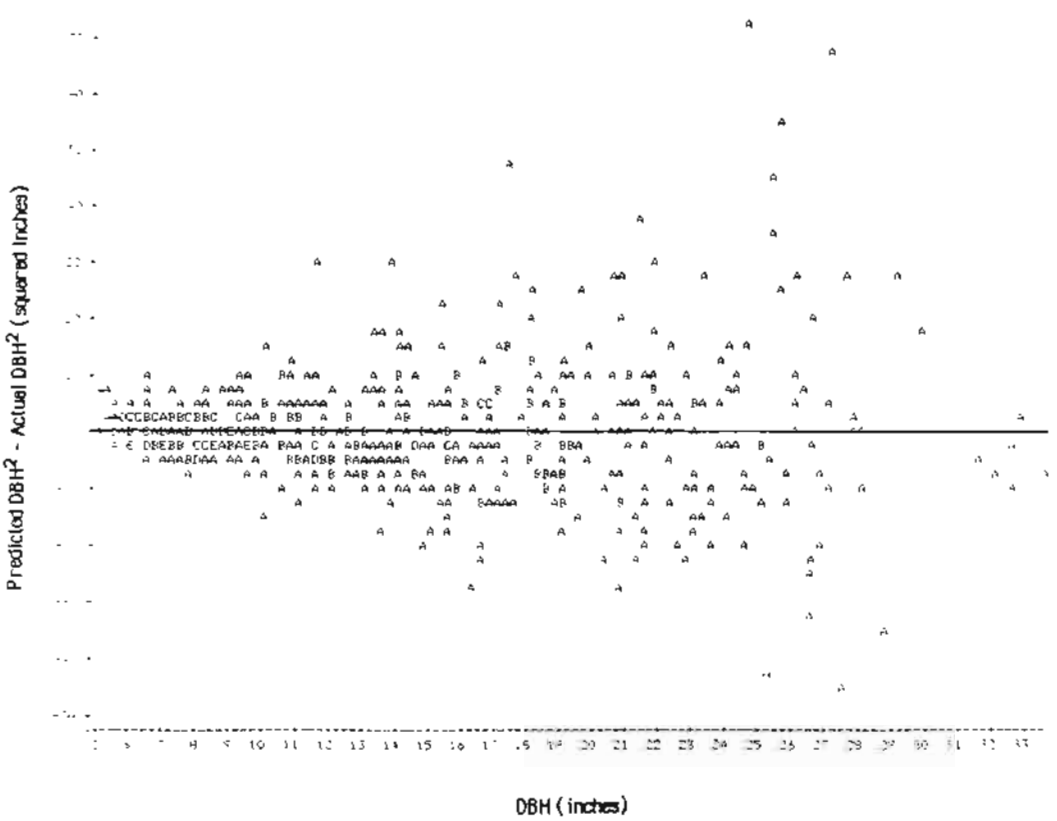

Figure 0.3 Residuals plotted on predicted diameter-squared growth--Douglas-fir.

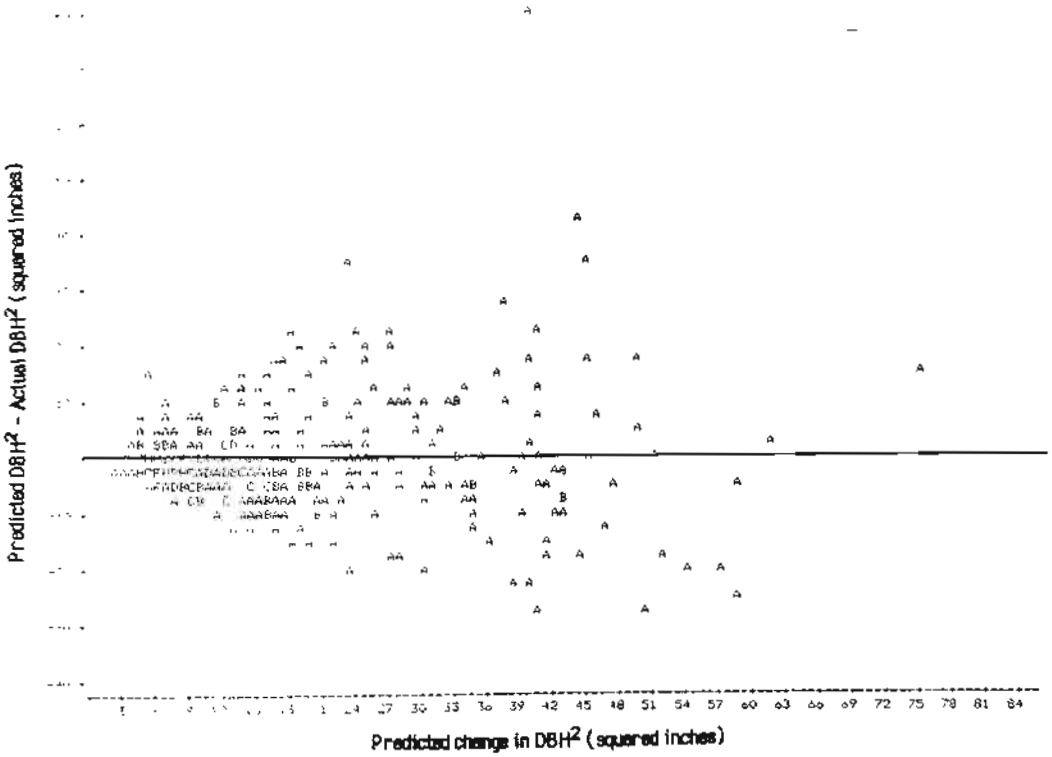

Figure 0.4 Residuals plotted on diameter--Douglas-fir.

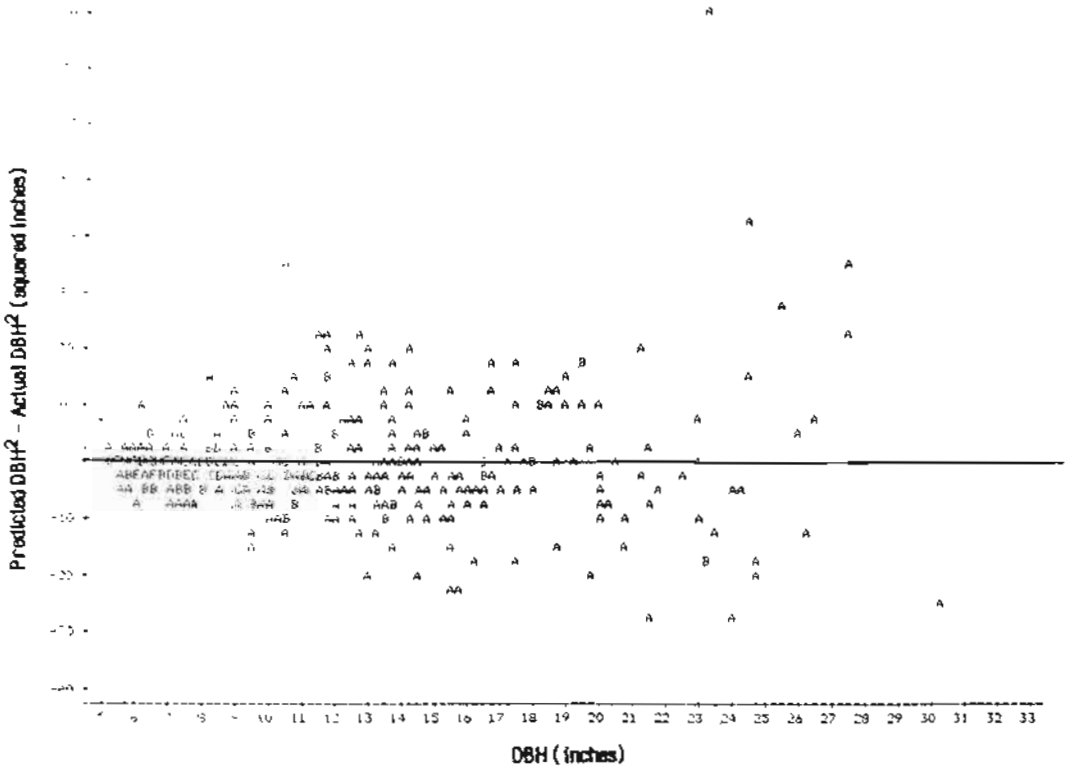

Figure D.5 Residuals plotted on predicted diameter-squared growth--white fir.

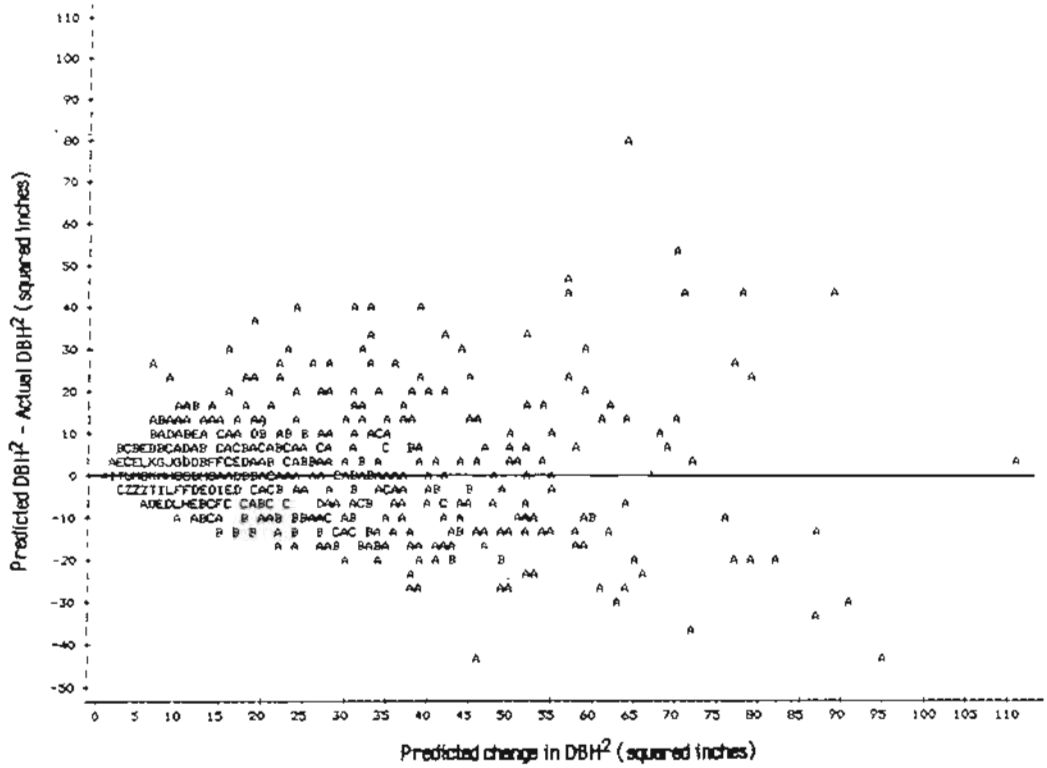
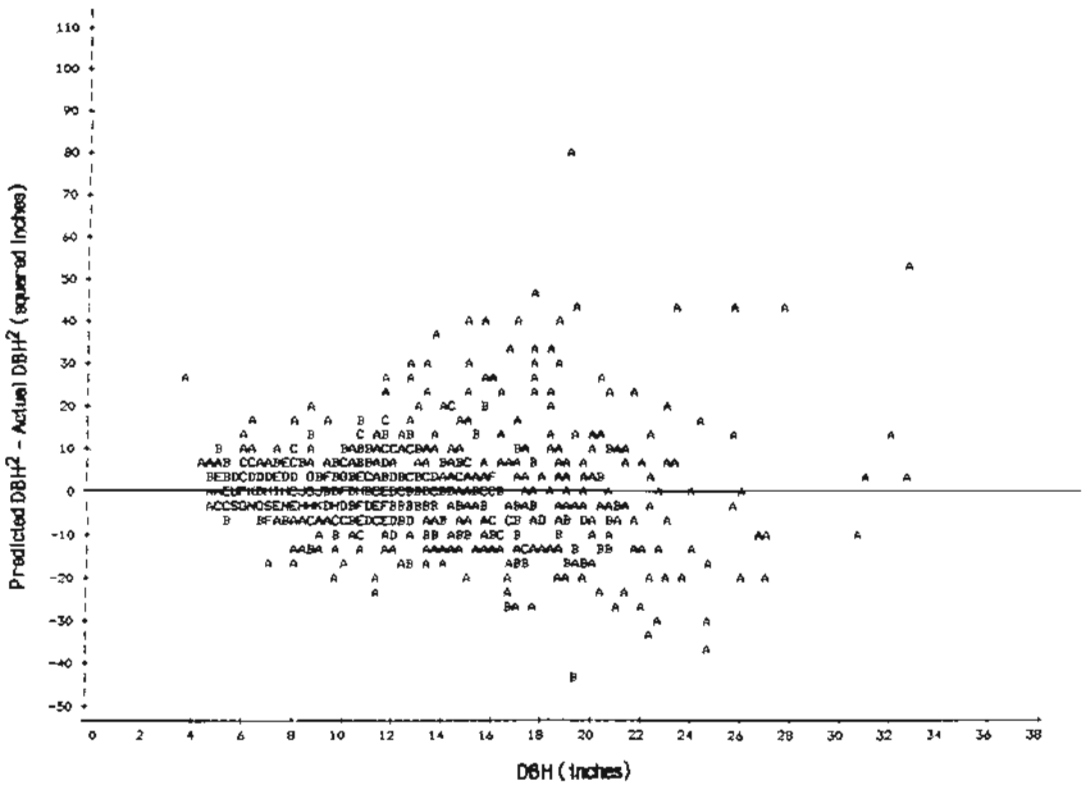
Figure D.6 Residuals plotted on diameter--white fir.