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HEIGHT GROWTH PATTERNS AND FIFTY YEAR BASE AGE SITE INDEX CURVES FOR YOUNG GROWTH COASTAL REDWOOD

by

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Abstract

Polymorphic site index curves with a fifty year breast-high age base were developed for dominant young growth redwood sprouts. Construction procedures utilized stem analysis data and height growth measured on permanent growth plots. The mathematical model used to generate these curves is also described. Cross checks against measured height growth of trees not used in developing these curves produced no indicators of bias. Similar checks against the Lindquist and Palley site curves revealed discrepancies in trees less than thirty years of age at breast height and in extreme site classes. The site curves developed here compare favorably with those of Lindquist and Palley only for older stands (over 30 years of age at breast-height) and for the intermediate site classes.

In view of these findings, the curves presented in this note will be used as the standard site classification basis for concurrent growth and yield studies being done for the region.

I. Introduction

This research note describes the results of studies of height growth patterns and site index estimation of young growth redwood. Site index is defined as the total height of dominant redwood sprouts at a breast-high age of fifty years. There were several reasons for undertaking this study: (1) statistical modeling of forest growth requires numerical rather than graphical relationships and a major objective of this study was to develop mathematical height-site index expression; (2) a fifty year base age is much closer to the probable future rotation age for young growth redwood in the region; (3) previous attempts by the authors to utilize Lindquist and Palley (1959) site curves to predict height growth indicated substantial distortions in the younger age classes and (4) there is current controversy (e.g. Curtis et al, 1974) involving the interpretation and specification of site index curves. The practical importance is that different methods of constructing height-site index models may give substantially different results and biased site estimation. Consequently, determining the magnitude of these differences was another prime objective.

Section II of this note is a discussion of possible sources of confusion in interpreting and using total height-site index relationships. Section III presents the new fifty year age base height-site index curves developed in this study and a comparison with the old curves of Lindquist and Palley. Data sources, statistical methods, and resultant mathematical expressions are included in an appendix for those interested in procedural aspects.

II. Interpretation and Uses of Height-Site Index Curves

Site Index models are usually presented as a sheaf of curves portraying the relationship between age and total height of dominant (or dominant and codominant) trees with the site index for each curve being the total height at some base age. In contemporary forest productivity research and application, these curves are used in three principal ways:

1) They are used to estimate site indexes which in turn are used as explanatory variables in developing equations to predict yield and growth of stand components other than height.

2) They are used to estimate site index to access the prediction equations developed in (1) above in application to specific stands.

3) They are used directly to estimate height growth of the stand component for which the curves are based.

Methodologically, the functional relationship on which height-site index curves are based can take either of two forms:

Height = f(Age,Site Index) (EQ.1)

Site Index = f(Age,Height) (EQ.2)

Equation 1 is a height growth expression which produces a direct estimate of height at a given age if site index is known and an indirect estimate of site index if age and height are known. The opposite is true for equation 2 which produces a direct (and we think more technically "correct") estimate of site index. The bulk of the literature on site index estimation and all of the conventional 'guide curve' methods of site curve construction utilize a form represented by Equation 1.

If height at various ages and site index were perfectly correlated, both equation forms would produce identical curves. This is seldom true and Curtis et. al. (1974) have shown that fitting both equations to certain data sets produces substantially different curve sets. This phenomena is due to statistical methodologies used in curve fitting rather than any faults of the trees.

<u>Site index estimates.</u>

In uses (1) and (2) outlined above, estimates of site index are never directly needed. The use of site index is confined to its explanatory ability in equations predicting the growth of stand components other than height. We maintain that, in this case, either equation 1 or 2 can be used to estimate site index and will result in no direct bias in the growth equations so long as one equation or the other is consistently used in model development and applications. In instances where methods of determining site index result in biased estimates (e.g. site index is correlated with age) growth models using site index as an explanatory variable would necessarily have to incorporate additional variables accounting for stand age at the time of site index determination.

Practical Considerations in Height-Site index models

For practical purposes, what is desirable is one age-height-site index expression that is used consistently to produce site index estimates in both growth model development and subsequent applications and is also capable of being used directly to produce estimates of height growth of acceptable accuracy under usual "less than perfect" situations.

Experimentation with both a height growth model and a site index model revealed the following (see appendix): 1). Beyond ages of 20 to 30 years there is almost no difference between the models in terms of predicted heights or site index. 2) Differences occur mainly in the very low and high sites between ages 10 and 20.

As site index estimates for stands less than ten years of age are quite unreliable regardless of the method employed, a height growth model was chosen as the specific functional form in this study because it is easier to use in modeling.

III. Height-Site Index Curves for Young-growth Redwood

A mathematical model was derived relating total height of dominant redwood sprouts to breast high age and site index (total height at a breast-high age of fifty years). The model and development are given in the appendix. Curves were generated from this model and are shown graphically in figure 1. Numerical values are listed in table 1. Figure 2 shows the relationship between the present curves and those of Lindquist and Palley. There is remarkable agreement between both sets of curves in the 30 year plus age range and in the site classes most abundant in the region. Discrepancies are largely in the younger age classes.

Site Curve Tests:

Both the height growth curves developed in this study and those of Lindquist and Palley were tested to see if there were biases resulting from improper methods of determining site index or poor specification of curve shape.

Measurements of total height and breast-high age taken at two different times over intervals of 10 to 15 years were available from 142 dominant redwood sprouts. These measurements came from permanent plot and stem analysis records not used in constructing the height-site index curves. The initial measurements on each tree were used to estimate site index. The resulting site value was then used to predict height at the second measurement and the following ratio computed:

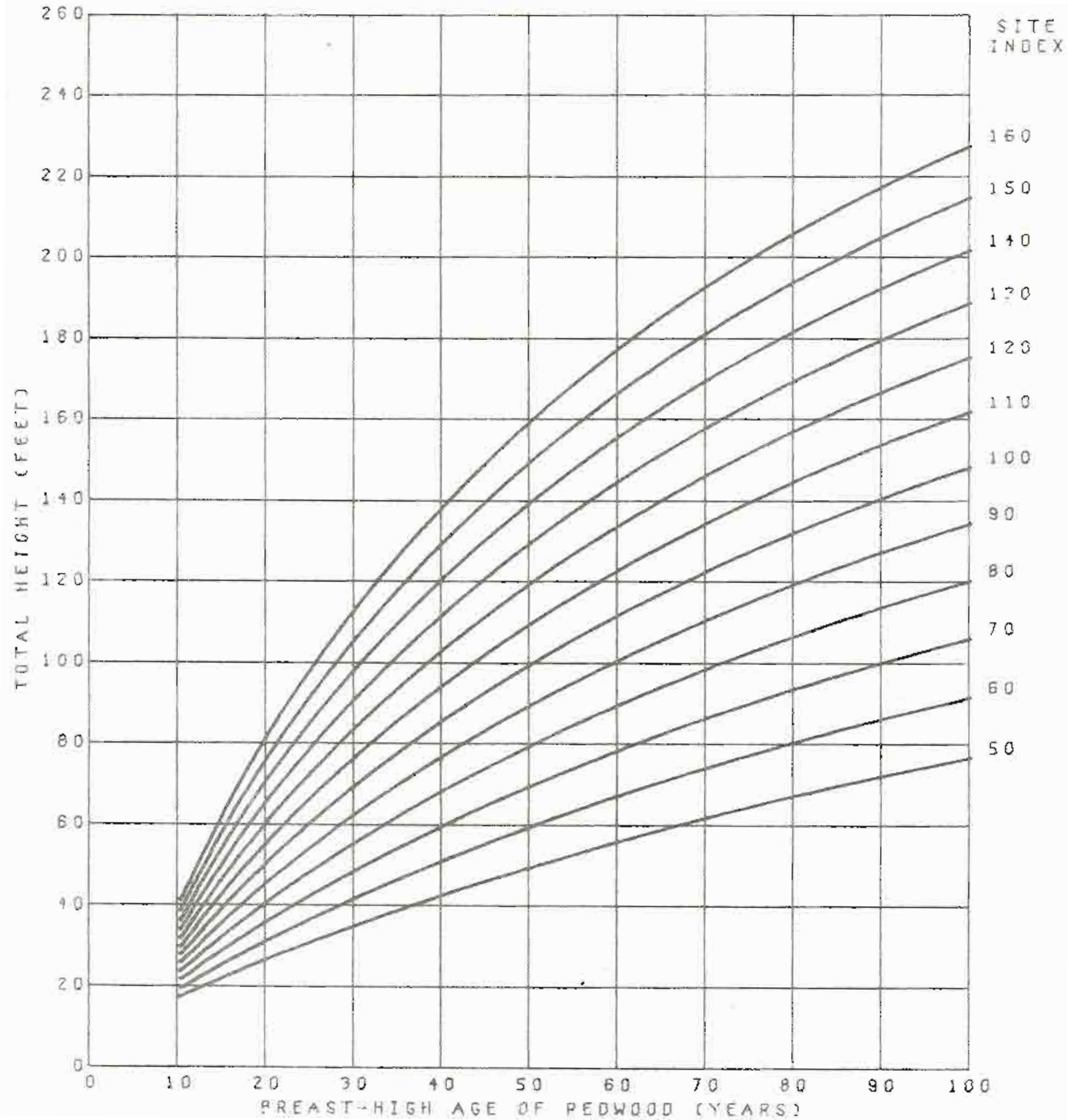

Figure 1. Fifty Year Age Base Site Index Curves for Dominant Young-Growth Redwood Sprouts.

Table 1. Average Total Heights of Dominant Redwood Sprouts by breast high age and site index.

BH	REDWOOD SITE INDEX											
AGE	50	60	70	80	90	100	110	120	130	140	150	160
10	17	20	22	24	26	28	30	32	34	37	39	42
12	19	22	25	28	30	33	36	39	42	45	48	51
14	21	25	28	31	35	38	41	45	48	52	56	60
16	23	27	31	35	39	43	47	51	55	59	63	68
18	25	30	34	38	43	47	51	56	61	66	70	75
20	27	32	37	41	46	51	56	61	66	72	77	83
22	29	34	39	45	50	55	61	66	72	78	84	89
24	30	36	42	48	53	59	65	71	77	83	90	96
26	32	38	44	50	57	63	69	76	82	89	95	102
28	34	40	47	53	60	67	73	80	87	94	101	108
30	35	42	49	56	63	70	77	84	92	99	106	114
32	37	44	51	59	66	73	81	88	96	104	111	119
34	39	46	54	61	69	77	85	92	100	108	116	124
36	40	48	56	64	72	80	88	96	104	113	121	129
38	42	50	58	66	75	83	91	100	108	117	126	134
40	43	52	60	69	77	86	95	104	112	121	130	139
42	44	53	62	71	80	89	98	107	116	125	134	143
44	46	55	64	73	83	92	101	110	120	129	138	148
46	47	57	66	76	85	95	104	114	123	133	142	152
48	49	58	68	78	88	97	107	117	127	136	146	156
50	50	60	70	80	90	100	110	120	130	140	150	160
52	51	62	72	82	92	103	113	123	133	143	154	164
54	53	63	74	84	95	105	116	126	136	147	157	167
56	54	65	75	86	97	108	118	129	139	150	161	171
58	55	66	77	88	99	110	121	132	142	153	164	175
60	56	68	79	90	101	112	123	134	145	156	167	178
62	58	69	80	92	103	114	126	137	148	159	170	181
64	59	70	82	94	105	117	128	139	151	162	173	184
66	60	72	84	95	107	119	130	142	153	165	176	187
68	61	73	85	97	109	121	133	144	156	167	179	190
70	62	75	87	99	111	123	135	147	158	170	182	193
72	63	76	88	101	113	125	137	149	161	173	184	196
74	64	77	90	102	115	127	139	151	163	175	187	199
76	65	78	91	104	116	129	141	153	166	178	190	201
78	67	80	93	105	118	131	143	156	168	180	192	204
80	68	81	94	107	120	133	145	158	170	182	194	206
82	69	82	95	108	121	134	147	160	172	184	197	209
84	70	83	97	110	123	136	149	162	174	187	199	211
86	71	84	98	111	125	138	151	164	176	189	201	214
88	72	85	99	113	126	139	152	165	178	191	203	216
90	72	87	100	114	128	141	154	167	180	193	205	218
92	73	88	102	116	129	143	156	169	182	195	207	220
94	74	89	103	117	131	144	158	171	184	197	209	222
96	75	90	104	118	132	146	159	172	186	199	211	224
98	76	91	105	119	133	147	161	174	187	200	213	226
100	77	92	106	121	135	149	162	176	189	202	215	228

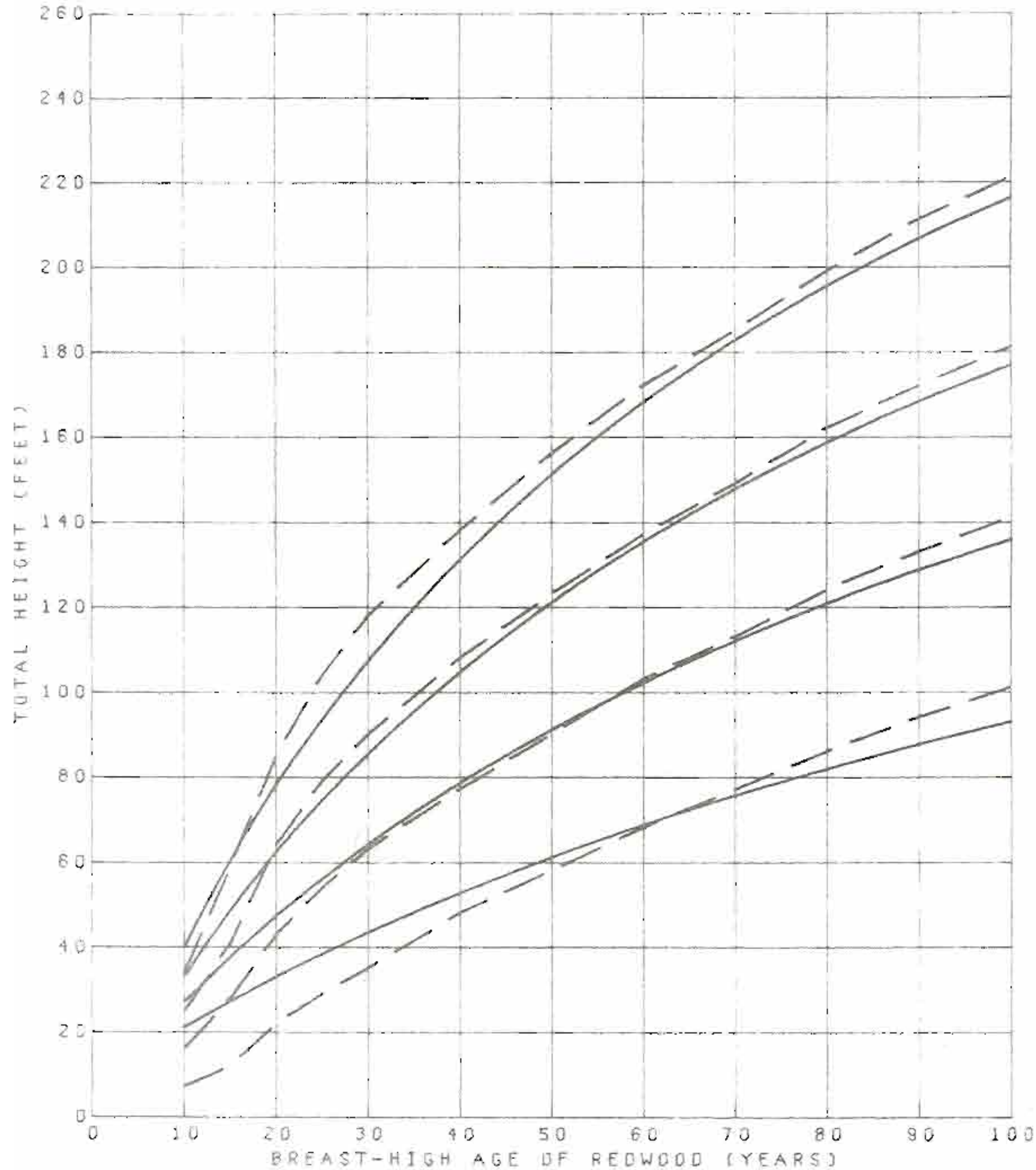

Figure 2. Comparison of curve shapes of fifty year base age redwood site index curves (solid line) with Lindquist and Palley (1961) site index curves for dominant young-growth redwood sprouts (dashed line)

$$x = \frac{\text{Total Height(Actual)} - \text{Total Height(Predicted)}}{\text{Age(terminal)} - \text{Age(Initial)}}$$

These ratios were separated into nine groups; three age groups denoted by the subscript 'i'(10-30 years, 30-60, and 60+) within three site groups denoted by the subscript 'j'(less than 90, 90-120, and 140+ for the 50 year base age curves and less than 140, 140 - 180, and 180+ for the Lindquist and Palley 100 year base age curves).

If there were no distortions in curve shape and no serious biases introduced by inverting the height growth model to obtain site index, we would expect the average of each group of ($\bar{x}_{ij}$) to be zero. Student's 't' ratios were computed for each group as

$$t_{ij} = \frac{\bar{x}_{ij}}{\text{standard error of } \bar{x}_{ij}}$$

and probabilities of obtaining larger 't' values were determined. These values are shown in table 2 and are further segregated by counties. While no hypothesis were formally tested, conventional statistical tests would favor concluding that significant differences exist between actual and predicted heights in situations where probabilities are .05 or less. It appears that the sample data fit the curves developed in this study better than they fit the Lindquist and Palley curves.

The possibility was raised by several cooperators that height growth patterns of redwoods might be different between Humboldt and Mendocino counties due to differences in climate or genetic provenances. The comparisons made here however do not reveal any material differences.

A parallel check involving a stepwise regression of various polynomials, cross products and transformations of age and site on values of 'x' computed for each observation produced the following results.

<u>50 year base curves</u>

$x = -.70 + .006(\text{Site index})$ $\qquad R^2 = .014$

<u>Lindquist and Palley Curves(LP)</u>

$x = -1.79 + .36(\text{Site index})$ $\qquad R^2 = .395$

Table 2. Probabilities of obtaining larger 't' ratios in comparing actual with predicted heights for fifty year base curves and Lindquist and Palley site curves.

50 Year Base Age Curves — Lindquist And Palley Curves

-All Counties-

Age	Site Group			Age	Site Group		
Group	90<	90-120	130+	Group	140<	140-180	180+
10-30	.32	.24	.19	10-30	.01	.01	.01
30-60	*	.79	.55	30-60	.72	.36	.87
60+	*	.40	.63	60+	*	.39	.82

-Mendocino County-

Age	Site Group			Age	Site Group		
Group	90<	90-120	130+	Group	140<	140-180	180+
10-30	*	*	.97	10-30	*	*	*
30-60	*	.16	.76	30-60	*	.84	.77
60+	*	.40	.69	60+	*	.39	.96

-Humboldt and Del Norte Co-

Age	Site Group			Age	Site Group		
Group	90<	90-120	130+	Group	140<	140-180	180+
10-30	.45	.24	.12	10-30	.01	.01	.01
30-60	*	.88	.33	30-60	*	.28	.67
60+	*	*	.77	60+	*	*	.58

* probabalities not computed if less than three samples available

These results would indicate that the differences between predicted and actual height growth are correlated with site index in a systematic fashion with the LP curves (F-ratio = 91.5). However, the correlation was not significant (F-ratio = 2.04) for the fifty year base-age curves developed in this study.

Conclusions

Although the curves developed in this study have suffered somewhat from an incomplete and limited data base, they do appear to offer a better fit in the younger age classes than the old curves and are directly usable in equation form for computer modelling. These curves can be improved as better stem analysis data becomes available.

Literature Cited

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Appendix A

I. DATA SOURCES

Stem Analysis. Records from 177 felled dominant redwood sprouts with ring counts and cumulative height measurements at fixed intervals along the tree bole were available for analysis. This data had been collected between 1898 and 1967 by a variety of personnel in Del Norte, Humboldt, and Mendocino counties. These measurements were converted to give a set of paired breast-high age and total height observations for each tree.

Permanent Plot data. Observations from 53 permanent plots having at least four dominant redwood sprouts measured for breast-high age and total height at least twice over a fifteen to twenty five year time span were also available. Observations were taken between 1952 and 1976 on plots maintained by members of the Redwood Yield Research Cooperative in Del Norte, Humboldt and Mendocino counties.

II. PRELIMINARY METHODS

The data in its initial form presented some problems. The stem analysis data section cuts were at fixed intervals thereby making heights as actually recorded unsuitable for direct use in statistical modelling. The data also lacked section cuts at the exact places to give heights at fifty years for site index measurements. The permanent plot data was also incompatible with the stem analysis data in its initial form. Lastly about 45% of all data sets did not have age measurements that spanned the proposed site index base age of fifty years.

In view of these shortcomings, the following procedures were considered to be most appropriate:

(1) The age-height data from each tree and each plot (data sets) were individually fitted to each of the following functions.

$$H = \frac{A^2}{(a+bA+cA^2)} + 4.5$$

$$H = a + \frac{b}{A} + cA + dA^3 + 4.5$$

$$H = e^{(a + \frac{b}{A})}$$

where A = breast high age and H = total height in feet

(2) All three functional forms were machine plotted against actual observations for each data set. Sets with obvious errors in measurement or recording were discarded.

(3) Standard errors and reasonableness of fit through the age range of each observation set were examined next and the best equation was retained to describe each data set.

(4) For data sets whose age, range did not span the site index base age the equations were further examined for reasonableness of fit when extrapolated to fifty years. If the extrapolation appeared unreasonable, the set was discarded. Tree or plot data sets with maximum ages less than 30 years or minimum ages greater than 70 years were discarded outright.

(5) These procedures left 123 stem analysis trees and 37 permanent plot records for further analysis. The equations for each data set were used to estimate site index (total height at fifty years breast high age) and to generate paired age-height observations at 10 year intervals throughout the age range of each observation set. No observations were generated for ages less than 10 or greater than 80 years of age. This gave a total of 559 observations of breast-high age, total height and site index. The numbers of samples by age and site groups are shown in table A-1.

Table A-1. Numbers of Observations by Age and Site Index Group

Site Group	Breast High Age(years)								
50 Yr Base Age	10	20	30	40	50	60	70	80	Total
71-80	3	4	5	4	4	3	3	3	29
81-90	3	5	5	5	5	2	1	1	27
91-100	6	8	10	12	12	9	8	6	71
101-110	10	14	14	15	15	13	9	8	98
111-120	25	29	31	34	37	30	23	20	224
121-130	5	6	8	12	12	12	10	10	75
131-140	4	4	5	5	5	4	4	4	32
Total	54	78	78	87	87	73	58	52	559

III. MODEL SPECIFICATIONS

After considerable experimentation with several possible model forms, a modified version of the sigmoidal exponential function described by Richards (1959) was used to express total height (H) as a function of breast high age (A) and site index (S). The form used in this study was

$$H = a_1 S^{a_2} \left[1. - \left(1. - \frac{S}{a_1 S^{a_2}}\right)^{a_5 S^{a_6}} e^{(A-50.)a_3 S^{a_4}} \right]^{\frac{1}{a_5 S^{a_6}}}$$

.....Equation A-1.

where the constants, a_1, a_2, ...a_6, estimated by non linear least squares, are as follows:

a_1 = 9.4366	R^2 = .988
a_2 = .68174	$S_{y.x}$ = 4.1 feet
a_3 = .0011842	Observations = 559
a_4 = .46112	
a_5 = .63885	
a_6 = .14567	

This equation is conditioned to predict a total height equal to the site index when the breast high age is fifty years. The form of this equation is general enough to allow for the expression of polymorphic curve patterns and variants of this model have been successfully applied to several other species (Payandeh, 1974).

IV. TESTING THE MODEL

In view of the somewhat piecemeal procedures that were necessary in developing this model, several tests were made to check for possible biases.

Model Specification. Even though the form of this model was considered to be extremely flexible, there is some possibility that it may not adequately describe the height growth patterns of redwood. To test for bias due to model specification, residuals from the fitted model were partitioned by ten year age classes and subsequently fitted to third degree polynomials of site index. The procedure was then reversed; third degree polynomials of age were fitted to residuals partitioned by 10 foot increments of site index. In no case was there sufficient evidence to suggest a functional misfit.

Differences due to Data Sources and Procedures. Three separate covariance analyses were made to see if there were differences that could be attributed to the following sources.

1) Differences in location -- Del Norte and Humboldt county data versus Mendocino county data,

2) Differences between stem analysis and permanent plot data and

3) Differences between data sets that required extrapolation to obtain site index and those that didn't.

In no case were there any statistically significant differences although tests of differences between counties were inconclusive because of dissimilar residual variances. However, testing the curves against height growth of trees from the individual counties (see text) did not reveal any bias.

V. HEIGHT GROWTH CURVES VERSUS SITE INDEX CURVES

A site index model was also fitted to all 559 observations. The form of the equation was the same as equation A-1 only the height and site variables were reversed. Payendah(1974) has shown that this produces curves of acceptable shape. Table A-2 shows the difference in predicted heights between the height growth model and the site index model.

Table A-2. Differences in predicted heights in feet between the height growth model and the site index model.

B. H.	Site Index						
Age	80	90	100	110	120	130	140
10	*	10	4	0	-4	-7	-10
20	4	2	1	0	-2	-3	-5
30	2	1	1	0	-1	-1	-2
40	1	1	0	0	0	0	0
50	0	0	0	0	0	0	0
60	-1	0	0	0	0	0	0
70	-1	-1	0	0	0	0	0
80	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0

* not computable

Height differences occur mainly in the younger age classes and beyond 30 years, there is no practical difference. The general relationships indicated in this table are similar to those found by Curtis et. al.(1974) but of a much lesser magnitude.