Estimating the accuracy of volume equations using taper equations of stem profile

GREG S. BIGING

Department of Forestry and Resource Management, University of California at Berkeley, Berkeley, CA 94720, U.S.A.

Received October 5, 1987

Accepted April 18, 1988

BIGING, G. S. 1988. Estimating the accuracy of volume equations using taper equations of stem profile. Can. J. For. Res. **18**: 1002–1007.

Comparisons of log volume estimation techniques are performed using the equations of Smalian, Huber, and Newton, and a numerical technique using cubic splines. The data utilized were obtained by predicting diameters at various points along the stem from two paper equations for white fir. Results indicate that Newton's and Huber's equations were the most accurate, followed by the cubic spline and Smalian's equation, respectively. This technique facilitated partitioning of the total error in volume estimation into measurement error and error due to model misspecification arising when the taper of logs could not be exactly described by a simple model such as a frustum of a paraboloid. For the taper relationships analyzed it was shown that the error due to the selection of an inappropriate mensurational model is less than 5% for a measurement distance of 16 ft (4.9 m) for all models tested and can be substantially reduced by applying the formulae only to logs positioned above basal swell. Systematic measurement error was assessed analytically and found to range between 1 and 4%. Thus, total error in volume estimation was less than 9% for all methods tested.

BIGING, G. S. 1988. Estimating the accuracy of volume equations using taper equations of stem profile. Can. J. For. Res. **18** : 1002–1007.

Des comparaisons de diverses techniques d'estimation du volume de billes ont été faites à partir des équations de Smalian, de Huber et de Newton et d'une méthode numérique faisant appel à des clés cubiques. Les données ont été obtenues par la prédiction des diamètres à divers points le long de la tige d'après deux équations de défilement pour le Sapin blanc. Les résultats montrent que les équations de Newton et de Huber ont été les plus précises, suivies par la clé cubique et l'équation de Smalian, respectivement. Cette méthode a facilité la répartition de l'erreur totale dans l'estimation du volume en erreur due aux mesures et en erreur due aux spécifications erronées du modèle découlant du fait que le défilement des billes ne pouvait pas toujours être décrit avec exactitude par un modèle simple comme le tronc d'un paraboloïde. À partir des relations de défilement analysées, on a démontré que l'erreur due au choix d'un modèle mensurationnel inapproprié est moindre qu'environ 5% pour une distance mesurée de 4,9 m pour tous les modèles expérimentés, et cette erreur pourrait être réduite de façon marquée en n'appliquant les formules qu'aux billes placées au-dessus du renflement du pied de l'arbre. L'erreur systématique due aux mesures a été évaluée de façon analytique et on a trouvé qu'elle se tenait entre 1 et 4%. Ainsi, l'erreur totale dans l'estimation du volume était inférieure à 9% quelle que fut la méthode expérimentée.

[Traduit par la revue]

Introduction

In determining volumes of logs, there are two major sources that contribute to the total error in volume estimation. First, there is the choice of the equation used in determining volume. The accuracy of a volume equation's prediction depends upon the underlying shape of the log (geometric solid). For example, if the shape of the log exactly follows the form of a frustum of a paraboloid, then the equations of Newton, Huber, and Smalian all provide exact results (Husch et al. 1982).[1] However, as the form of the log departs from parabolic, then the formulae of Huber and Smalian become biased, and if the form radically departs from parabolic, then Newton's formula becomes biased (Wensel 1977[2]). When this situation occurs, using these formulae will introduce bias into volume estimation. This component of total error is termed model misspecification error. A second source of error is introduced when diameters and lengths of logs are not accurately measured.

If volume is being determined via water displacement there can be other sources of measurement error, if water adheres to the log, for example, or if the log is not submerged it must be held down by mechanical arms. Both these factors can affect displacement. As an example of measurement error associated with water-displacement determination of volume, Martin (1984) found that the coefficient of variation based on repeated readings of the xylometer for logs containing 5 ft^3 (0.14 m^3) or more was under 1%, but as the size decreased to 3–5 ft^3 (0.08–0.14 m^3), the coefficient of variation increased to 2–3%.

Studies of the accuracy of mensurational formulae for assessing volume have previously been conducted using two different approaches. These approaches either compare estimated volumes with "true" volumes calculated when the measurement interval is small (approximately 2 ft (0.6 m)), or compare estimated volumes with "true" volumes determined by water displacement. In either case, true volumes are unknown as there are measurement errors associated with either method and errors associated with the selection of a mensurational formula. Thus, for these studies the results are not readily interpretable because they are confounded with several sources of error.

[1]Newton's equation is given as $V = (A_1 + 4A_3 + A_2)\, L/6$, Smalian's equation as $V = (A_1 + A_2)\, L/2$. and Huber's equation as $V = (A_3)\, L$, where V is cubic-foot volume, A_1 and A_2 are the cross-sectional areas of the ends of the log (ft^2), A_3 is the cross-sectional area of the log midpoint (ft^2), and L = log length (ft)

[2]L.C. Wensel. 1977. A generalized prismoidal log volume equation. Biometrics Note No. 5. Mimeographed publication of the Department of Forestry and Resource Management, University of California, Berkeley.

Printed in Canada / Imprimé au Canada

Using the water-displacement technique, Young et al. (1967) compared volumes of northern hardwoods and softwoods in Maine estimated using the formulae of Smalian and Huber with values obtained by displacement when logs were immersed in a xylometer. They found that for 8- and 16-ft (2.4 and 4.9-m) logs the average errors associated with Huber's equation (3.5% for 8-ft (2.4-m) logs and −3.7% for 16-ft (4.9-m) logs) were consistently smaller and statistically different from the average errors obtained with Smalian's equation (≈9.0% for 8- and 16-ft (2.4 and 4.9-m) logs). As the log length decreased to 4 ft (1.2 m), the errors in volume estimation decreased, and there were no significant differences between the volumes estimated with Huber's and Smalian's equations. Martin (1984) compared volumes of 243 eastern hardwood logs (12.3 ft (3.7 m) in length) estimated with 14 different equations, including those of Smalian, Huber, and Newton, and compared them with values obtained by water displacement of logs. Martin found that Huber and Newton's equations performed the best in predicting cubic volume, followed closely by Smalian's equation. The biases associated with Huber's, Newton's and Smalian's equations were 2.5, 3.9, and 6.9%, respectively. The mean "true" log volume for the 243 logs was 6.1 ft^3 (0.17 m^3).

Another approach in evaluating volume formulae is to compare the tree volume predictions (obtained by summing the volumes predicted for logs) with the best available prediction of tree volume. The best prediction is considered to be the sum of the log volumes calculated when the measurement intervals are as small as possible. For example, Goulding (1979) examined the accuracy of several standard mensurational formulae and a spline function[3] in estimating tree volumes when the interval between measurements varied. The errors obtained as a percentage of the volume calculated from using a small measurement interval (approximately 2 ft (0.6 m)) varied by method and distance between measurements. On the average, Goulding found that a spline curve had an error that was 60% of the error obtained using Smalian's method. Newton's equation had an error that was, on average, 50% of the error associated with Smalian's method. When the distance between measurements was less than 6.6 ft (2 m) all the methods tested had small errors (less than 2.3%). However, as the interval between measurements increased, the percent error of tree volume calculated with Smalian's method increased rapidly. At a distance of 16.4 ft (5 m) the percent errors were 8% for Smalian's equation, 5% for the spline equation, and 4% for Newton's equation. At a distance of 9.8 ft (3 m) the errors were less than one-half the errors obtained at 16.4 ft (5 m).

Thus, assuming an 8- to 16-ft (2.4- to 4.9-m) log, these research results indicate that the total errors encountered in estimating volume are approximately 3–9% for Smalian's equation, 3–4% for Huber's equation, 1–4% for Newton's equation, and 2–5% for the cubic spline equation. Because of the rather large errors reported in studies of the accuracy of Smalian's equation, Husch et al. (1982) recommend that this equation not be used unless the logs are in 4-ft (1.2-m) lengths. However, because these results include measurement error, the effect due to selection of a volume formula is overestimated.

Methods

The primary objective of this paper is to separately evaluate the sources of error in volume estimation (model misspecification error and measurement error) by utilizing taper equations to represent stem profile. With traditional mensurational techniques the evaluation of sources of error in volume estimation cannot readily be performed. This is because true volumes are never known and estimates of "true" volumes are confounded with measurement and model misspecification error, whether water-displacement techniques or traditional mensurational formulae are used to estimate log volumes.

Hence, an alternative approach, which supplements traditional methods, is taken using statistical models of tree taper (profile) to provide "exact" diameters at points along the tree stem. With this technique, known volumes can be obtained by integration of the profile equation. Because diameters are specified without error, the confounding of sources of error can be eliminated. Hence, the effect of model choice on the accuracy of volume estimation (termed model misspecification error) can be examined in the absence of measurement error. The effect of measurement error on volume estimation can be estimated separately and these two components combined to form the total error in volume estimation. The results are compared with previous research findings.

This approach, however, has an inherent postulate that taper equations accurately portray the form of logs and trees. This premise is plausible but not entirely warranted. Relatively accurate profile equations have been developed for many tree species. (cf. Biging 1984; Bruce et al. 1968; Demaerschalk and Kozak 1977; Goulding and Murray 1976; Kozak et al. 1968; Max and Burkhart 1976), but these profile equations display some bias at various relative heights above ground. If, however, the relationship is sufficiently close, this approach will realistically portray the influence of measurement error and model misspecification error on true tree volumes.

Estimation of model misspecification error

Error in volume estimation due to model misspecification was analyzed by investigating the effect of volume formula[4] and log length (distance between measurements) over 25 size classes for white fir, *Abies concolor* (Gord. & Glend.) Lindl. (lowiana (Gord.)) taken from Biging (1984). The 25 size classes investigated were composed of 2 in. diameter classes from 10 to 30 in. (25.4–71.1 cm) and 20-ft height classes from 50 to 130 ft. (15.2–39.6 m) primarily falling along the main diagonal of the diameter–height stand table taken from Biging (1984). Cubic volumes were estimated for each tree of a given size class by summing the volume calculated for each section[5] and were compared with the "actual" cubic volumes obtained by integrating a sigmoidal taper model (eq. 1) and a segmented polynomial taper model (eq. 2). The results presented are averaged over the 25 size classes. The taper equations and coefficients values can be found in Table 1.

Biging (1984) compared the diameter predictions of model [1] with those of model [2] developed by Max and Burkhart (1978) and judged by Cao et al. (1980) to outperform other models tested in terms of bias, standard error, and mean absolute deviation. Biging found that models [1] and [2] compared closely in performance as judged by standard error of the estimate. The degree of

[3]A spline function uses a set of polynomial segments with smooth joins to create a smooth curve between specified data points (see Liu 1980).

[4]Note that the volume of the tip of the tree was computed under the assumption that the tip was conical in shape.

[5]For Newton's equation the log lengths were twice the length of the logs used with the other equations. This insured that the distance between measurements was the same for all methods used. This is necessary because Newton's equation requires three measurements of the diameter of each log, whereas the other equations require only one or two measurements.

TABLE 1. Coefficient values for taper equations [1] and [2] for white fir, *Abies concolor* (Gord. & Glend.) Lindl. (lowiana (Gord.))

The sigmoid-derived taper equation is given as

$$[1] \quad d = \mathrm{DBH}\ \{b_1 + b_2 \ln[1 - (\frac{h}{H})^{1/3} (1 - e^{-b_1/b_2})]\}$$

where

- d = diameter inside bark (in.) at a point h ft above the ground
- H = total height (ft)
- DBH = diameter at breast height (in.)

Values of coefficients are as follows:

- b_1 = 1.0933
- b_2 = 0.3643

The segmented polynomial model is specified by

$$[2] \quad d^2 = b_1\mathrm{DBH}^2(\frac{h}{H} - 1) + b_2\mathrm{DBH}^2(\frac{h^2}{H^2} - 1) + b_3\mathrm{DBH}^2(a_1 - \frac{h}{H})^2 I_1 + b_4\mathrm{DBH}^2(a_2 - \frac{h}{H})^2 I_2$$

where

- $I_i = 1$ if $\frac{h}{H} \le a_i$; i = 1, 2
- $I_i = 0$ if $\frac{h}{H} > a_i$
- d, H, and DBH defined as in [1]

Values of coefficients are as follows:

- b_1 = −2.6788
- b_2 = 1.2778
- b_3 = −1.7449
- b_4 = 75.0475
- a_1 = 0.5850
- a_2 = 0.0734

bias was similar for both models, although the levels varied by relative height and species. However, he found that both models well represented tree form. Hence, the profiles predicted using taper equations [1] and [2] are assumed to be sufficiently close to actual tree profiles to allow an accurate partitioning of the components of error in volume estimation into the effect of measurement error and model misspecification error. Models [1] and [2] are both used as a basis for generating log diameters without measurement error to investigate model misspecification error. A comparison of the results from these two models will demonstrate the sensitivity of results to the underlying taper surface generated by each model.

Estimation of measurement error

Measurement errors were assessed algebraically (see Analysis and results) by computing the ratio of volumes predicted with the mensurational formulae, assuming consistent measurement error, to volume computed assuming no measurement error. These two sources of error were combined to form the total error in volume estimation and compared with previous research findings.

Analysis and results

The total error in volume estimation comprises measurement error and model misspecification error. These sources will be separately analyzed and compared with the results of previous studies.

Impact of measurement errors on the accuracy of volume estimation

The impact of measurement error on cubic volume estimates can be obtained algebraically by allowing diameters and lengths of logs to vary within specified limits. For Smalian's equation the ratio of volume that includes measurement error to volume estimated without error is given as follows:

$$\frac{S_\Delta}{S} = \frac{K\ [(D_1 + \Delta D_1)^2 + (D_2 + \Delta D_2)^2]\ [L + \Delta L]}{K(D_1^2 + D_2^2)L}$$

where

- S = Smalian's cubic foot volume without measurement error
- S_Δ = Smalian's cubic foot volume with measurement error
- $K = \frac{\pi}{(2 \times 4 \times 144)}$
- ΔL = log length measurement error
- D_1, D_2 = the diameter measurements at the ends of the log (in.)

After some algebraic calculation, and assuming that the terms involving ΔD_i^2 (i = 1, 2) and $\frac{\Delta L}{L(D_1^2 + D_2^2)}$ are negligible, then

$$\frac{S_\Delta}{S} \approx 1 + \left[\frac{(2D_1\Delta D_1 + 2D_2\Delta D_2)}{(D_1^2 + D_2^2)}\right] + \frac{\Delta L}{L}$$

To estimate the mean effect of measurement error, a Monte Carlo simulation approach could be employed after making distributional assumptions about measurement errors. However, for this study we need only know the ranges in values due to measurement errors to facilitate comparison with previous studies. A worst-case analysis will allow establishment of ranges within which we can expect the effect of measurement error to be bounded. The greatest measurement error arises when there is a consistent bias (positive or negative) in measurement. Assuming that $\Delta D_1 = \Delta D_2 = \Delta D$, it follows that

$$\frac{S_\Delta}{S} \approx 1 + \left\{\frac{[2\Delta D(D_1 + D_2)]}{[D_1^2 + D_2^2]}\right\} + \frac{\Delta L}{L}$$

The relative percent change in volume estimation of logs due to measurement error is given by

$$[3] \quad 100\left[\frac{S_\Delta}{S}\right] - 100 \approx 100\left\{\frac{[2\Delta D(D_1 + D_2)]}{[D_1^2 + D_2^2]} + \frac{\Delta L}{L}\right\}$$

Using a similar derivation for Newton's equation the relative percent change in volume estimation of logs due to consistent measurement error is given by

$$[4] \quad 100\left[\frac{N_\Delta}{N}\right] - 100 \approx 100\left\{\frac{[2\Delta D(D_1 + 4D_3 + D_2)]}{[D_1^2 + 4D_3^2 + D_2^2]} + \frac{\Delta L}{L}\right\}$$

And for Huber's equation the relative percent change in volume estimation of logs due to measurement error is given as

$$[5] \quad 100\left[\frac{H_\Delta}{H}\right] - 100 \approx 100\left\{\frac{2D_3\Delta D_3}{D_3^2} + \frac{\Delta L}{L}\right\}$$

Assuming a log length of 16 ft (4.9 m), length measurement error of 1/10 ft (0.03 m), diameter measurement error of 1/10 in. (0.25 cm), and a taper of 1 in. (2.54 cm) in 8 ft (2.4 m), then the percent errors vary from approximately 4.5% for a 6 in. (15.2 cm) diameter log to approximately

FIG. 1. The average percent errors in cubic volume estimation of trees as the distance between measurements varies, based upon a sigmoid-derived taper equation (model 1).

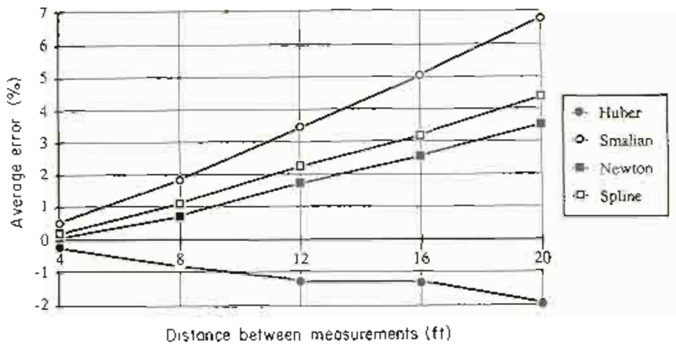

FIG. 2. The average percent errors in cubic volume estimation of trees as the distance between measurements varies, based upon a segmented polynomial taper equation (model 2).

1.3% for a 30-in. (76.2-cm) log.[6] There is only 1.8% error in volume for an 18 in. (45.7-cm) diameter log. It is quite interesting to note that the results of this analysis are almost identical for all three mensurational formulae. Thus, it appears that the effect of consistent measurement errors affects these equations equivalently.

The values vary little as the stated assumptions are relaxed. That is, for log lengths of 8, 12, and 16 ft (2.4, 3.7, and 4.9 m) and for taper rates of 0.8, 1.0, and 1.2 in. (2.0, 2.5, and 3.0 cm) per 8 ft (2.4 m), measurement errors do not change markedly. Assuming an error of 1/10 in. (0.25 cm) in diameter measurements may be conservative, and thus more allowance may be justified. Under these circumstances, allowing the measurement error to double results in approximately a doubling in the error in volume estimation, because the approximation to error is directly proportional to the magnitude of diameter measurement error[7] (see eqs. 3, 4, and 5). The measurement errors could be negative (underestimating diameter) and the effect would be to underestimate volume. In this case, this would simply change the sign of the values. The case of partially countervailing errors, while not investigated, would tend to lessen the effect of measurement error.

For this study, tree volume is estimated by summing the volumes associated with each log (segment) of the tree. As trees are composed of logs of varying diameters, the error due to measurement should be weighted by the relative volume of the tree accounted for by each log. As the values are relatively stable over a wide selection of diameters, the error in tree volume estimation can be approximated by choosing a range of values that encompasses the log size classes of interest.

Impact of model misspecification error on the accuracy of volume estimation

Error in volume estimation due to model misspecification was analyzed by investigating the effect of volume formula and log length (distance between measurements) averaged over the 25 size classes.[8]

Newton's equation

Wensel (see footnote 2) has shown that Newton's equation is exact when taper can be expressed as a third-order polynomial (hereafter termed third-order polynomial form (TOPF)), which is a function of distance from one end of the geometric figure (log). However, when the profile of the log cannot be expressed in TOPF, Newton's equation does not provide exact results. Because models [1] and [2] are complex and cannot be expressed in TOPF, it follows that Newton's equation will not provide exact results. Figures 1 and 2 display the results of the average percent error in tree volume estimation (averaged over the 25 tree size classes) as a function of distance between measurements obtained by applying Newton's formula to the "exact" diameter and height values predicted with models [1] and [2]. It is evident that for any distance between points investigated (4–20 ft (1.2–6.1 m)), Newton's equation is virtually unbiased for taper equation [1]. Even at a distance of 20 ft (6.1 m), the bias was less than 0.5%. However, for taper equation [2], Newton's equation was positively biased for all measurement intervals. At a distance of 8 ft (2.4 m), bias was under 1%, at 16 ft (4.9 m) it was 2.5%, and at 20 ft (6.1 m) it was 3.5%.

If 1–4%, at most, is added for measurement error then the total error in tree volume estimation is in the range 1.5–4.5%, based on eq. 1 or 3.5–6.5% based on eq. 2, for a measurement interval of 16 ft. (4.9 m). This is similar to the results obtained by Martin (1984) and Goulding (1979). Thus, it appears that the total error (measurement error and model misspecification) is in the range 1.5–6.5% when Newton's equation is used. Considering only the error due to model misspecification, results from this study confirm that when there are departures from form for which Newton's equation is exact, volume estimation is biased when standard log lengths are used. For both models tested, there was less than 2.5% model misspecification error in estimating tree cubic volume for a standard log length of 16 ft. (4.9 m).

[6]The approximating formulae presented in eqs. 3, 4, and 5 predict values within 98–100% of values calculated with the full expansion.

[7]Errors in measuring log length exert only a minor effect on volume estimates. For a measurement error of 0.1 ft on a 16-ft log, the term 100 $(\Delta L/L)$ is only 0.6 and thus contributes only slightly to the calculated values.

[8]The effect of size (DBH) is not presented herein because, with a minor exception, it had no discernible influence on model misspecification error. The exception to this trend occurred in some of the smaller diameter classes in the range 10–14 in. for several of the volume formulae for which there was a small increase in relative percent error in these classes.

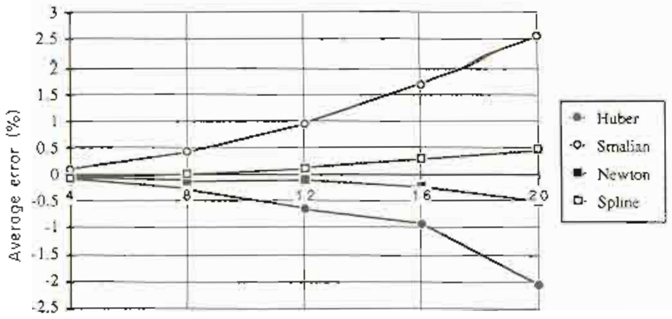

FIG. 3. The average percent errors in cubic volume estimation of trees as the distance between measurements varies. Estimates are based upon a sigmoid-derived taper equation (model 1), beginning at breast height to exclude basal swell.

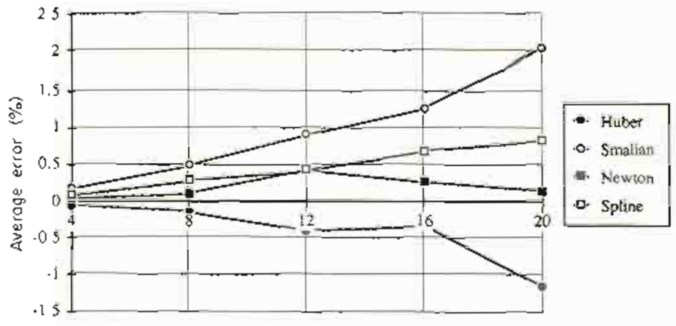

FIG. 4. The average percent errors in cubic volume estimation of trees as the distance between measurements varies. Estimates are based upon a segmented polynomial taper equation (model 2), beginning at breast height to exclude basal swell.

Smalian's and Huber's equations

For any solid it can be shown algebraically that the error associated with Smalian's equation is twice that of Huber's, and of the opposite sign to that associated with Newton's equation (Husch et al. 1982). If the geometric solid is an exact frustum of a paraboloid then both formulae yield exact results. As the log form departs from parabolic, then the volumes predicted from both Huber and Smalian's equations become biased. As taper departs from TOPF, then Newton's equation is no longer exact, but the algebraic relationship between the errors associated with Huber's and Smalian's equations holds relative to Newton's equation for equivalent log lengths. This cannot be seen directly from Figs. 1 and 2 because results are plotted for intervals between measurements, not log length. However, for both taper models tested, the error relative to Newton's equation for Smalian's and Huber's equations is given by a factor of −2 when log lengths are equivalent.

In Figs. 1 and 2 it can be seen that the average percent errors for these two methods are less than 3.5% for either taper model for distances up to 12 ft (3.7 m). As the distance increases beyond 12 ft (3.7 m), the average percent differences increase rapidly for Smalian's equation, which has an average overestimate of 2.7 and 5.1% for models [1] and [2] at 16 ft (4.9 m) and 3.7 and 6.8% for models [1] and [2] at 20 ft (6.1 m), respectively. At 20 ft (6.1 m) Huber's equation underestimates, on average, by less than 2% and was approximately 1% at 16 ft (4.9 m) for both models. It is interesting to note that because Huber's equation is a function of midlog cross-sectional area, it was the least sensitive to changes in the underlying taper model.

If up to 1–4% is allowed for measurement error for Smalian's or Huber's equation, then the total error in tree volume estimation is approximately 4–7% for model [1] and 6–9% for model [2] for Smalian's equation, and 2–5% for models [1] and [2] for Huber's equation at a measurement distance of 16 ft (4.9 m). This differs only slightly from Martin's (1984) and Goulding's (1979) results. Considering only model misspecification error, results for both taper models show less than 5% error in estimating tree cubic volume with Smalian's equation and less than 2% error with Huber's equation for a standard log length of 16 ft (4.9 m).

Spline functions

Spline functions have been successfully used to model taper of individual trees (cf. Lahtinen and Laasasenaho 1979; Liu 1980; Goulding 1979) and to calculate log cubic volumes. For a mathematical formulation of cubic spline functions see, for example, Burden et al. (1979). The spline function for taper can be integrated directly to yield cubic volume. Figures 1 and 2 display the average bias of the spline function as distance between measurements increases for models [1] and [2], respectively. For this study, the spline approximation never exceeds an average of 2% for any distance between data points for eq. 1, but for eq. 2 the spline errors were 3.2% at 16 ft (4.9 m) and 4.4% at 20 ft (6.1 m). Excluding the 4-ft (1.2-m) distance, the average ratio of spline error to Smalian volume error was 57% for eq. 1 and 62% for eq. 2, which is almost identical with the results of Goulding (1979).

Assuming the same range of errors in volume estimation as occurred for Newton's equation (1–4%), then the total error in tree volume estimation is less than 3–6% for eq. 1 and approximately 4–7% for eq. 2 at 16 ft (4.9 m), which is in the range of errors that Goulding (1979) reported. If only model misspecification error is considered, the bias in tree volume estimation is less than 3.5% for both models considered for a standard log length of 16 ft (4.9 m).

Effect of basal swell on model misspecification error

For taper profiles developed from eqs. 1 and 2 there are only subtle differences in the taper profiles.[9] However, the segmented polynomial taper equation (model 2) exhibits more basal flare than the sigmoid-derived taper equation (model 1) and may account for a significant proportion of the differences in volume accuracy between these two models. To test this hypothesis, models [1] and [2] were reanalyzed using only predicted ("exact") diameters at 4.5 ft (1.4 m) and above to remove the effect of basal swell. The results are presented in Figs. 3 and 4.

When Figs. 1 and 3 are compared, the reduction in model misspecification error for the various mensurational formulae judged against model [1], which has less basal flare than model [2], is not pronounced. For Smalian's equation there was only a reduction a 1% at 16 ft (6.1 m). However, when Figs. 2 and 4 are compared for model [2], there were dramatic reductions in model misspecification error for all mensurational models examined. At 16 ft. (6.1 m) the errors were less than one-third the values displayed in Fig. 2 and did not exceed 1.5% for any mensurational model. These

[9]Equation 1 predicts more volume in the lower portion of the tree bole and is a more "regular" profile than that predicted with eq. 2.

results imply that the effect of model misspecification is greatest in the basal log in which departures from TOPF are common and is only moderate for logs from all other parts of the tree.

Conclusions

The total error in log (and tree) volume estimation has two components, one due to measurement error and one due to model misspecification arising when the underlying shape of the log departs from a specified geometric shape such as a frustum of a paraboloid. An alternative approach, which supplements traditional techniques, was taken using profile equations to provide "exact" diameters at points along the tree stem under the premise that well-constructed taper equations are representative of tree and log profiles. Unlike traditional techniques, this construct allows the two sources of error to be separately assessed.

The results of this study indicate that the errors in tree cubic volume estimation for white fir resulting from model misspecification for the four methods tested (Smalian's, Newton's, and Huber's equations and a spline function), while substantial, are less than expected for some models and are affected by basal swell. When measurements are taken at a distance of 20 ft (6.1 m), all models tested had an average error of less than approximately 7%, and less than 5% at 16 ft (4.9 m). Below 12 ft (3.7 m) there was minor error associated with all four methods (<3.5%). Newton's and Huber's equations fared best, but Newton's was biased as taper departed from third-order polynomial form. As expected, Huber's equation outperformed Smalian's. The tree volume estimation errors associated with Smalian's equation were smaller than expected, averaging less than 5% for a standard log length of 16 ft (4.9 m). The cubic spline function and Huber's equation performed very similarly, with Huber's equation being about 90% or less of the value of the error associated with the cubic spline. Huber's equation, which is a function of midlog diameter, was the least affected by differences in the underlying taper surfaces tested. Thus, it appears that all of the methods tested provide relatively accurate estimates of cubic volume for standard log lengths.

For one taper equation, which exhibited a higher degree of basal flare, the accuracy of the estimates of cubic volume was notably increased for Smalian's and Newton's equations and for a spline equation when applied to logs above breast height. This result showed that basal swell can have a relatively large influence on the accuracy of the volume estimates. Therefore, when using this technique particular care should be taken to select a taper equation that realistically portrays lower stem profile. It also follows that a considered choice of a mensurational model for the basal log is warranted to minimize model misspecification error.

It was estimated that consistent errors in measurement (over- or under-estimation of 1/10 in. (0.25 cm) in diameter, and log length estimation errors of 1/10 ft) resulted in 1–4% change in volume estimation. Therefore, for this study the total error in volume estimation ranged from approximately 2 to 9%, depending on method and distance between measurements and the severity of measurement errors.

This technique was shown to provide results in concert with previous research findings, and also allowed estimation of the error associated with the choice of a mensurational formula. This methodology is less expensive than traditional techniques and is easily modified to allow for additional analysis that would be difficult to achieve without conducting additional experiments, such as assessing the effect of a different set of log lengths on the accuracy of volume formulae.

Acknowledgments

This research was supported in part by the Northern California Forest Yield Cooperative and McIntire-Stennis Project 3679-MS. Special thanks go to Dr. Ronald L. Giese for providing computing facilities and support while the author was on sabbatical leave at the University of Wisconsin — Madison. Thanks are also extended to Peter J. Daugherty for his assistance in computer programming and processing of the data during the first phase of this project.

BIGING, G.S. 1984. Taper equations for second-growth mixed conifers of Northern California. For. Sci. **30**: 1103–1117.

BRUCE, D., CURTIS, R.O., and VANCOEVERING, C. 1968. Development of a system of taper and volume tables for red alder. For. Sci. **14**(3): 339–350.

BURDEN, R.L., FAIRES, J.D., and REYNOLDS, A.C. 1979. Numerical analysis. Prindle, Weber and Schmidt, Boston, MA.

CAO, Q.V., BURKHART, H.E., and MAX, T.A. 1980. Evaluation of two methods for cubic-volume prediction of loblolly pine to any merchantable limit. For. Sci. **26**(1): 71–80.

DEMAERSCHALK, J.P., and KOZAK, A. 1977. The whole-bole system: a conditioned dual-equation system for precise prediction of tree profiles. Can. J. For. Res. 7: 488–497.

GOULDING, C.J. 1979. Cubic spline curves and calculation of volume of sectionally measured trees. N.Z. J. For. Sci. **9**(1): 89–99.

GOULDING, C.J., and MURRAY, J.C. 1976. Polynomial taper equations that are compatible with tree volume equations. N.Z. J. For. Sci. **5**(3): 312–325.

HUSCH, B., MILLER, C.I., and BEERS, T.W. 1982. Forest mensuration. John Wiley and Sons, New York.

KOZAK, A., MUNRO, D.D., and SMITH, J.H.G. 1968. Taper functions and their application in forest inventory. For. Chron. **45**(4): 278–283.

LAHTINEN, A., and LAASASENAHO, J. 1979. On the construction of taper curves by using spline functions. Metsantutkimuslaitoksen Julk. **95**(8): 1–63.

LIU, C.J. 1980. Log volume estimation with spline approximation. For. Sci. **26**(3): 361–369.

MARTIN, A.J. 1984. Testing volume equation accuracy with water displacement techniques. For. Sci. **30**(1): 41–50.

MAX, T.A., and BURKHART, H.E. 1976. Segmented polynomial regression applied to taper equations. For. Sci. **22**(3): 283–289.

YOUNG, H.E., ROBBINS, W.C., and WILSON, S. 1967. Errors in volume determination of primary forest products. *In* Proceedings of 14th IUFRO Congress, Munich, September 4–9, 1967. Part VI, Sect. 25. pp. 546–562.