A Tree-Based Forest Yield Projection System for the North Coast Region of California

By

Bruce Edwin Krumland

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Chairman Date
Greg S. Biging 6/28/82
Kjell D. Jason 7/6/82

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Abstract

A yield projection system is developed for forest stands in the North Coast region of California. The anticipated end use required that the system be applicable to actual existing forest stands, be capable of accepting a wide range of harvest prescriptions, be compatible with the kind of information collected in conventional inventories, and can be calibrated to local conditions.

Consideration of these design requirements as well as a means of accounting for the impact of heterogeneous species and age compositions on stand yields led to the adoption of a tree-based distance-independent growth modelling approach. For each species, the model system is composed of four main equations that are applied to individual trees within a stand. These equations predict the probability the tree will die in the next five years and the five year change in tree diameter, total height, and height to the crown base.

Three important features of tree growth processes, as perceived in modelling, are identified and used as a basis for the statistical characterization of the model system and subsequent estimation of system parameters. These features are: 1) the system is a multiple equation system; 2) the system describes both cross-sectional (growth differences between trees) and time series (growth trajectories of individual trees) phenomena; and 3) the system operates as a recursive process. In part, a random coefficient regression approach is used to account for some of

the variation in growth between trees. Particular emphasis is given to the interpretability of the model system resulting from different methods of estimating coefficients. Two main model types are distinguished: a type A model, which describes the growth process that generated the current stand condition, and a type B model which describes the current growth relationships in existing stand conditions. A type A model form is most consistent with observed phenomena of forest growth and possesses properties that are desirable attributes of recursive growth models. Development of a type A model system was subsequently adopted as an analytical goal.

Several ad hoc procedures are developed to circumvent analytical problems attributable to the inadequate data base available for modelling. The model system is subsequently implemented and compared with other models and long term development records of sample plots. A reasonable compliance between simulated and actual stand growth was observed. Possible modifications and extensions of the the methods employed in this study are discussed.

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This dissertation is dedicated to my mother and father, who years ago, always hoped that their sons would continue in college and get an education they never had. In recent years, this hope was replaced by a dread that their offspring would all become tenured graduate students. Take heart mom, I'm done.

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Chapter 1

INTRODUCTION

This research will focus on the development of a forest growth and yield projection system that can be applied to mixed species stands in the North Coast region of California. Growth and yield projection systems in forestry are not new. Vuokila (1965) for example, reports a documented effort to relate volume yields to stand age that dates to 17th century China. However, the last twenty years has seen a rapid expansion in not only the types of forest stands analyzed but also in methodologies. As general features, forest growth and yield models, whether they are graphical, tabular, or mathematical representations, are designed to provide estimates of the future characteristics of forests. Methodologies and modelling approaches have subsequently evolved out of the interactions of several factors: 1) increased specificity of forecasting objectives, 2) data availability in quantifying strategic relationships, 3) costs and time limitations, 4) increased recognition of the validity in using mathematical and statistical methods as analytical devices, 5) increasing reliance on computers, both in the analytical phase of model development and as a computational aid in making growth and yield projections, and 6) specific characteristics of the forest resource under analysis.

In general, the interplay of these factors relegates the implementation of a growth and yield model to a specific forest resource a case study. From this viewpoint, the current research can be considered "case oriented". While currently there are enough similarities in published models to allow them to be segregated into several specific "methodological classes", even among these specific approaches, there is considerable variation in analytical procedures employed in model construction and operating conventions used in forecasting. It is this diversity that contributes to the current status of growth and yield modelling as being more an art form than a pure science based on a generally accepted unifying theory. Assuming that implemented models satisfy the objectives of forecasting, a pragmatic viewpoint would assert that a unifying consensus is unnecessary. That is, an acceptable end result justifies the means. It is the contention of the author however that there are still unresolved problem areas in the design and implementation of growth and yield models in forestry. This study will concentrate on the resolution and possible solutions of some specific problems associated with the type of methodology chosen for implementing a growth and yield projection system for coastal stands. In this context, this research can be considered a contribution to general growth and yield methodology using exemplifying data from forest stands in the North Coast of California.

1.1 Objectives and Justification

The major objective of this research is to design and implement a forest growth and yield projection system that can be applied to mixed species stands in the North Coast region of California. The model system is intended to be used as an operational timber management tool and is to have the following capabilities:

1) The system is to apply to actual on-the-ground timber stands

regardless of age types, species mixture, or past history. Often, growth models are developed for stands with specific characteristics. For example, a common focal point is the evenaged monoculture. However, limitations in stand applicability limits the resource base for which projections can be obtained. In forest-wide planning, projections are required for all stands and to limit the model to stands that meet certain specifications reduces the effectiveness of the system as a management tool.

2) As a <u>link in the forest planning system</u>, input data needed to drive the system should be compatible with the type of data collected in forest inventories. Similarly the output from the system (projections) essentially provides estimates of strategic variables needed in decision-making. Compatibility with the later process requires that the system be capable of providing estimates in the kind of units needed for subsequent analysis As growth projections are needed for a variety of different decision processes, the capability of tailoring the projection summary units to specific tasks is a desirable attribute.

3) The primary management variable of interest in this type of model is a <u>harvest capability</u>. The system should be capable of accepting a wide range of harvest prescriptions. This requirement ultimately translates in the ability to simulate the harvests of a variable number of trees by species, size class or other characteristics. Currently, management objectives of all possible users of this model and stand conditions are not so uniform as to consider only a small range of harvest possibilities.

4) The system should be capable of being "<u>calibrated</u>" or adjusted to stands with historical growth information particularly if the projections made by the model are at variance with past performance. It is somewhat heroic to expect that projections made by a model with extensive applicability would be precise and accurate in all circumstances. Thus, a calibration capability makes more effective use of available information and increases the resolution level at which the model operates.

These capabilities of the projection system form the basis for a practical and useful forest management tool. These model characteristics and features have not been arbitrarily chosen. They essentially constitute the working objectives of the Redwood Yield Research Cooperative under whose auspices this research was performed. This cooperative is composed of forest industries and state organizations whose members are charged with managing a large proportion of the coastal forest resource. It is reasonable to assume then, that these capabilities are keyed to the practical needs of potential users of the model.

The most obvious justification for implementing the system for the North Coastal region is that it fills a large gap in effective forest planning in the region. The only currently available yield projection models are the "normal" yield tables for Douglas fir (Schumacher, 1930) and redwood (Bruce, 1923) and "empirical" yield table for redwood (Lindquist and Palley, 1963). While the methodologies employed in constructing these models are often considered deficient by today's standards, more serious objections are that they are for undisturbed stands with very specific characteristics; even-aged monocultures meeting certain stocking requirements. To the extent that not all stands have these characteristics, their usefulness is limited. Lindquist and Palley (1967) also presented models to predict 10-year growth for even-aged, undisturbed predominantly redwood stands. While treating stocking as a variable increases the usefulness of this formulation, the projection is for a fixed time span and the limitations on stand conditions hampers its usefulness.

In summary, this research is justifiable because there is nothing available that is remotely capable of providing the type of growth and yield projection information needed at an operational level of forest management in the region.

1.2 Methodological Contributions

The approach chosen for implementing the coastal yield projection system uses growth of individual trees as the basic units of analysis without direct regard to their spatial orientations[1/]. This type of approach is sometimes referred to as a distance-independent tree-based growth model (Munro, 1974). The individual models of this type of approach usually have terms representing the impacts on individual tree growth due to competition from surrounding trees. For models with harvesting capabilities, competition terms are of vital concern because the growth response of residual trees after harvesting is logically modelled as a change in competition. A density index is developed in this research which has been found to provide a consistent and biologically related measure of competition.

A second methodological aspect dealt with in this research concerns

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1/ An overview of the possible approaches that are available and the rationale for choosing this are detailed in Chapter 2.

interpretation and possible biases in parameter estimates of component equations in the model system when fitted by conventional methods with the kind of data commonly used in constructing this type of a system. Identification and correction of model biases are a central concern in growth and yield models as they ultimately effect the predictions and the validity of the system. A source of bias common to many implemented models is identified, the effects are indicated, and possible corrections are discussed.

The last major methodological aspect addressed in this research is a consideration of techniques to calibrate the system to specific forest stands on the basis of past growth data. Growth models, particularly those that are intended to apply to a wide range of stand types and management practices, frequently have large prediction errors. When evidence exists to indicate the system over or under predicts, an efficient and logical extension of the model would be to include a procedure for calibration. With the exception of work done by Stage (1973) this aspect has received little attention in the literature.

1.3 Limitations in Scope

In any modelling effort of this type, some limitations in scope are inevitably required due to data and time constraints. The following restrictions in model applicability were found to be pragmatically necessary.

1.3.1 Species

There are over 30 tree species (Munz and Keck, 1959) found in the North Coast region of California. Most of these species have little commercial importance. Consequently, the following species groupings were recognized based on relative extent, commercial importance, and similarity in growth characteristics.

1) young growth redwood (Sequoia sempervirens)
2) young growth Douglas fir (Pseudotsuga menziesii)
3) other young growth conifers
4) tanoak (Lithocarpus densiflora)
5) red alder (Alnus rubra)
6) other hardwood species
7) old growth redwood
8) other old growth conifers

Old growth timber available for commercial purposes is confined to a small number of "virgin" stands or else occurs as scattered residuals in more vigorous young growth stands. Based on personal communication with several industrial foresters, the remaining commercial coastal old growth is expected to be effectively liquidated in the next two decades. Consequently, no attempt is made to model growth for old growth species. When present in young-growth stands however, they were considered to contribute to competition of young growth species.

The species groups of most interest in this study are young growth redwood, Douglas fir and tanoak. By far, the two conifers are the most abundant and commercially important species in the region. Tanoak, while also abundant is of much less commercial importance and is commonly referred to as a weed species. It is a management concern however, because it occupies growing space that could otherwise be devoted to more valuable conifers and affects their growth through competition. Hence, this study will focus on these three species. Limited evidence indicates that the "other conifer group" can be treated as Douglas fir in terms of modelling. The "other hardwood group" has limited data available and will be treated as tanoak. Alder, while being of minor importance is sufficiently different from the other groups to warrant individual treatment. However, at this time, the lack of data necessitated abandoning the complete implementation of the alder component in the model.

In summary, this study will focus primarily on redwood, Douglas fir, and tanoak as species components.

1.3.2 Size Limitations

Out of necessity, prudent forest managers are interested in growth and development of trees and stands throughout their entire life cycle; from seed burst to ultimate harvest. Mainly because of data limitations, the model developed in this study is applicable only to stands composed of trees roughly 4 or more inches (10.2 cm) in diameter, 4.5 feet (1.3 meters) off the ground outside bark (DBH) and 20 feet (6.2 meters) or larger in total height.

1.3.3 Ingrowth

In this study, ingrowth is defined as trees growing into the smallest size class recognized in modelling during a growth period. Ingrowth is sometimes a management concern, particularly when a timber stand is being managed on an uneven-aged basis which relies on natural seed fall for regeneration. A system component which attempts to predict ingrowth due to natural regeneration will not be developed. However, the model system developed here is flexible enough so that if some external basis for introducing ingrowth into the model is available, it can be facilitated.

1.3.4 Timber Volumes

A major item of managerial concern in forestry is the volume of wood that can be grown and harvested in a timber stand. Tree volumes however are seldom observed directly as it is a difficult, time consuming, and expensive task, particularly when trees are still standing. The "conventional" procedures are to develop models relating timber volumes to readily measurable tree characteristics such as tree DBH and total height.

This convention is adopted in this study. All prediction equations are for changes in measurable tree characteristics, notably tree DBH and total height. These two variables are then used to predict tree volumes. Existing tree volume equations (Krumland and Wensel, 1978b) have been adopted for use in this study.

1.3.5 Data Limitations

Growth modellers seldom have the luxury of designing a model system, identifying the variables needed to quantify strategic relationships, and completely procuring the appropriate base of observational data needed to ideally implement the system. Forest growth data are relatively expensive to collect and the adequate quantification of many growth relationships requires years or even decades of repeated observations on individual trees or plots. Out of necessity, growth modellers frequently resort to historical growth measurements as a primary data base. Data available for this study were assimilated from several sources in cooperation with members of the Redwood Yield Research Cooperative. These data were primarily composed of past measurement records of existing permanent growth plots. Between the years 1975-1979, supplementary measurements and evaluations were made in an attempt to insure a consistent and complete base of observational data. Still, many historical measurements such as past unmeasured tree heights and crown dimensions could not be easily reconstructed at the level of detail needed for an ideal analysis. Consequently, the types of methodologies and degree of analysis were constrained by the availability of existing data.

1.4 Study Overview

The remainder of this study describes the process followed in implementing a growth and yield projection system for North Coastal California. Chapter 2 describes the design and architecture of a yield projection system that meets the objectives of this study. Chapter 3 is devoted to theoretical considerations in model specifications and subsequent parameter estimation. Chapter 4 describes the development of a tree competition index. Chapter 5 presents the results of fitting the model system to data. Chapter 6 presents a preliminary implementation framework and evaluation of model performance. Chapter 7 is devoted to a consideration of calibration methodologies. Chapter 8 is a summary and recommendations for future work.

Chapter 2

GROWTH PROJECTION SYSTEM ARCHITECTURE

In this chapter, the rationale and architecture of a growth projection system that can fulfill the objectives stated in Chapter 1 is developed. The spectrum of currently available modelling philosophies is summarized and used as a basis for the design and description of a growth model system. A descriptive summary of characteristic features of the coastal forest resources is also provided as a fundamental consideration in formulating the system is to select a design that can conceptually encompass the range of stand conditions that are currently found or are likely to exist in future. Resolution of the model design with the study objectives is subsequently evaluated.

2.1 Growth and Yield Modelling Approaches

Since nomenclature in forest growth modelling is by no means unified, various modelling approaches cannot be rigorously classified. There are similarities as well as differences between all approaches. A classification scheme, however, involves the identification of characteristics from which an appropriate segregation can be made. In growth and yield modelling, there are several possibilities which might include a) a distinction as to whether the growth process was viewed as a differential, a finite difference, or an integral relationship, b) whether the model was based on theoretical or empirical relationships, c) output capabilities in terms of estimates of different classes of roundwood products, d) whether it operates deterministically or has stochastic features.

Following Munro (1974), a segregation of modelling philosophies based on the primary unit of analysis is illuminating from an operational viewpoint because it provides a basis for evaluating input requirements, output capabilities, and possibilities for simulating management actions. The following classification is not intended to be rigid as several published papers draw upon features in several different classes. It does however, provide a satisfactory basis for formulating a growth projection system.

2.1.1 Whole Stand Models

In this approach, the primary unit of analysis is the stand[1/] and characteristics of interest are usually some measure(s) of stand volume, basal area, numbers of trees, and average diameter with all values usually expressed on a per acre (hectare) basis. Frequently, these characteristics are based on trees larger than some minimal DBH and tree volumes are estimated on the basis of some merchantability standard (e.g. the volume between a fixed stump height and a certain minimum upper bole diameter). The innumerable historical works on "normal" and "empirical" stand yields can be considered "whole stand models" although the methodologies employed bear little resemblance to currently used procedures.

Modern whole stand models use some combination of the variables previously described, a measure of stand productivity (usually site index), and age if the stand is evenaged, to predict future growth. If the growth relationship is differential, integration gives estimates of

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1. More accurately, this approach should be referred to as a "whole plot model" as all of the modelling is done with plot data rather than measurements of entire stands.

future stand characteristics. If growth is expressed as a finite difference, repeated solution and summing produces these estimates. The most common means of quantifying growth relationships is via some form of regression analysis. This approach is prominent in growth studies of evenaged monocultures. Works by Buckman (1962) and and Sullivan and Clutter (1972) are examples of whole stand models applied to this stand type. Moser (1967) has also applied the whole stand approach to unevenaged forest conditions. Vuokila (1965) provides an extensive world wide literature review of stand modelling efforts.

2.1.2 Distributional Models

In models of this class, it is assumed that the frequency distribution of tree DBH's within the stand can be adequately represented by a continuous density function. Parameters of this density function are then related to stand attributes such as age (in evenaged stands), minimum, maximum, and average DBH, as well as measures of stand height, density, etc., usually through some form of regression analysis. Forecasts of these attributes are used to estimate future values of the parameters of the density function at some time 't'. If N_t is an estimate of the future numbers of trees, d is tree DBH, and $f(d,\theta_t)$ is the density function, then basal area between any two diameters d_1 and d_2 is given as

$$\left[cN_t\right]\int_{d_1}^{d_2} d^2 f(d,\theta_t)\delta d$$

where c is a constant.

Similarly, if tree volume at time t can be expressed as a function of tree diameter ($g_t(d)$), then substituting this function for d^2 in the previous expression and integrating gives an estimate of stand volume in the diameter range.

Distribution models have a higher degree of resolution than stand models because they allow estimates for distinct fractions of the whole stand. Diameter distribution models have been developed using Weibull density functions (i.e., Clutter and Allison, 1974, Lohrey and Bailey, 1976), beta functions, (Burkhardt and Strub, 1974), as well as empirically derived density functions (Goulding, 1972).

An alternative procedure that can be employed with distribution models is to derive estimates of the density function parameters directly from other stand parameters rather than use statistical based estimates. Stand modelling is used directly to estimate primary stand attributes such as trees per acre ($\hat{N}_t$) basal area ($\hat{B}_t$) and volume ($\hat{V}_t$). If, for example, θ_t is a three dimensional vector of density function parameters, then the following set of equations can be solved to obtain estimates of θ_t:

$$1 = \int f(d,\theta_t)\delta d$$

$$\hat{B}_t = (cN_t)\int d^2 f(d,\theta_t)\delta d$$

$$\hat{V}_t = N_t \int g_t(d) f(d,\theta_t)\delta d$$

This method insures that the sum of stand fraction estimates are compatible with estimates for the whole stand and does not require the added analysis of fitting prediction equations for θ_t. Hynick (1979) has described this method in more detail.

2.1.3 Stand Table Projection Methods

In the traditional form of this method (Spurr, 1952), numbers of trees of a given species in an arbitrary diameter class within a stand are the basic units of analysis. Sampling provides estimates of past growth rates of each class which usually take the form of simple averages. Projections are made under the assumption that future growth rates are the same as past rates at least for short projection periods and if no harvesting and limited mortality occurs. Under these conditions, the method is fairly reliable but of limited utility. Ek (1974) has extended this method to estimate changes in numbers of trees by diameter class as a function of diameter class competition, mortality, and harvests.

2.1.4 Tree Models

In this approach individual trees within stands (plots) are the basic units of growth analysis. Future growth of individual tree characteristics such as tree DBH, total height, and crown dimensions are often expressed as a function of current tree size characteristics, a site productivity measure, and a measure of competitive stress which is some aggregate function of characteristics of the surrounding trees. A recurrent theme in tree increment research with direct applications to modelling is the "open grown" tree concept. Tree growth is thought of as being being effected by two multiplicative factors: a) a potential which predicts the level of growth of a individual tree in the absence of other trees or in "open grown" stand conditions and b) a competition term which scales the growth of a tree from "1" in open grown conditions toward "0" as a tree begins to become overtopped in dense stand. This concept seems to have been satisfactorily applied in several modelling efforts (e.g., Ek, 1974, Mitchell, 1975, Daniels et. al. 1979) and was formalized as a general growth "law" in the early 1900's (Baule, 1917). Individual tree age is seldom used however, in even aged stands where all trees are approximately the same age, stand age is sometimes used. Dudek and Ek (1980) have assimilated a bibliography of over 40 tree based forest growth models. There are two main types of tree models: (1) distance dependent tree models and (2) distance independent tree models.

In the former type of model, trees are also identified by their planar coordinates within a stand. The chief purpose of using coordinates is to construct a competition index for the subject tree on the basis of its relative size and proximity to other trees. The method consequently has the potential of accounting for the impacts of stand spatial heterogeneity on individual tree growth.

For distance independent tree models, spatial coordinates are not recognized and competition indices are developed independently of tree spatial arrangements within a plot.

2.2 Characteristics of the Coastal Forest Resource

The study area extends from northern Sonoma County in the south northward to the Oregon border. Sampling has been confined mainly to areas where coast redwood occurs. In this region, this is primarily in the zone of coastal fog influence which usually is a strip that seldom exceeds thirty miles from the coast but in several situations, extends much farther up river drainages.

While there are numerous tree species found in this area, the stands of major interest are those composed of young growth redwood, Douglas fir, and tanoak in various mixtures. Largely as a result of historical clear cutting of old growth stands, many current young growth stands are even-aged. Redwood components in these stands are mainly of sprout origin although redwood is also capable of reproducing from seed.

Douglas fir in these stands are of seed origin while tanoak is both a vigorous seeder and sprouter.

On alluvial benches, redwood frequently occurs in pure stands. Further up slopes, Douglas fir becomes a common associate. Developmental histories of these mixed stands are interesting, particularly from a modelling stand point. In the first growing season after logging, redwood sprouts from stumps and frequently attains a height of 6-7 feet (Fritz, 1959). Drawing on an already established root system, subsequent growth in DBH and height, particularly of dominant sprouts, is quite vigorous. Redwood and Douglas fir seedlings also become established in the first few years after logging. Depending on the timing of natural regeneration and the degree of brush invasion which may hamper early seedling growth, trees of seed origin may take anywhere from 6 to 20 years to reach 6-7 feet in height. Subsequent development of these stands depends on several factors. First there is usually a site index differential between species. In a mixed species stand, a typical site index mix (50 year age base) might be tanoak - 85 feet, redwood - 110 feet, and Douglas fir - 130 feet. This phenomenon usually results in the more vigorous Douglas fir trees catching up in height with the sprouting redwood 40-60 years after logging and surpassing them thereafter. Second, if there is an abundance of redwood sprouts, this differentiation takes much longer as the sprouting redwood are a significant source of competition to the trees of seed origin in the understory. Conversely, relatively greater numbers of Douglas fir collectively act as a major source of competition on redwood in older stages of young-growth stand development.

Response to competition also differs between species. Redwood is relatively tolerant and able to endure heavy levels of competition for extended time periods and still respond favorably to release. Douglas fir trees however are more shade intolerant than redwood and grow less under heavy levels of competition. Tree mortality rates are also much higher for Douglas fir than redwood. Tanoak in mixed stands is similar to redwood in tolerance. In young and sparsely stocked stands, tanoak frequently takes on the appearance of a large bush.

Progressing up slopes and inward from the coast to drier environments, the proportion of redwood in stands decreases and other conifers, tanoak, and associated hardwoods become more prevalent. Throughout the study area, residual old growth trees left during earlier logging are scattered in various concentrations throughout some of these stands.

Partial harvesting of second growth coastal stands has occurred largely as a post World War II phenomenon, in response to dwindling supplies of old growth timber. Some of these harvests resemble classical thinnings from below: removal of most of the smaller suppressed trees and spacing control to enhance the growth of the residual stand. However, other stands have had harvests which removed large portions of dominant and codominant stand fractions. These harvests were historically concentrated in Douglas fir because of price premiums relative to redwood. Impacts on the resulting species composition of these stands were diverse. When partial harvests were severe, conditions were favorable for the establishment of an understory. Redwood sprouted vigorously and other conifers seeded in. Frequently, this practice resulted in a mass infusion of tanoak, particularly in the drier interior regions. The resulting stands were consequently multi-tiered. Lastly, there are managers who are deliberately attempting to create stands of multiple age classes with the idea of eventually managing them under some form of uneven aged management.

In summary, the young growth coastal forest resource is diverse and dynamic in terms of species composition and size and age structure. The major species occur in both mixtures and monocultures of even and mixed age stands, many of which have been modified by historical harvesting activities.

2.3 Choice of Approach

The modelling approaches described above encompass the spectrum of approaches currently applied in forest growth modelling. Whole stand and distributional models were not considered to be viable alternatives in the design of a coastal growth projection system. Whole stand models have been successfully applied to evenaged monocultures and to some mixed-species uneven aged stand conditions where the growth characteristics of individual species is similar. However, attempts to model mixed species stands that explicitly recognize growth differences between species have been confined to limited stand conditions. Turnbull (1963) modelled the growth of mixed evenaged uncut Douglas fir-alder stands. Extensions to include several species in stands of indeterminate age and structure would seem to require unmanageable complexity or consider loss in resolution relative to tree model approaches. Distributional approaches are also considered to be of limited utility. Diameter distributions in coastal stands may or may not be well defined. Harvesting may noticeably alter the basic shape of the underlying size distribution and truncate the distribution from either tail. In these situations, it becomes conceptually easier to model the growth of individual trees rather than the stand as a whole.

The system adopted here can be classified as a distance independent tree model approach. While the stand table projection method in the mode postulated by Ek (1974) might at least form a conceptual basis for modelling, it was felt that only modelling DBH increment was too restrictive for our needs. The addition of tree height to DBH in a tree volume prediction equation significantly improves the precision of individual tree volume estimates. Consequently, tree height and necessarily, height increment were considered to be desirable system components. Also, tree crown size is biologically related and statistically highly correlated with tree growth. Crown size is a good integrating variable representing the effects of past stand history on the potential of a tree for future growth: trees that have developed in dense stand conditions tend to have small live crowns which limits their capacity for future growth. This is particularly important in predicting the effects of thinning operations on trees in the residual stand. Hence, tree crown size was also felt to be an important tree characteristic that should be modelled.

The use of tree coordinates, while conceptually forming a highly resolute basis for deriving individual tree competition factors, was felt to be of limited utility because: a) it requires stem mapping of plots which is an additional expense, b) it requires additional computer space and computation time in making projections and c) it requires a special type of plot configuration (fixed area) in inventory procedures and d) available evidence (Johnson, 1970) suggests that the added efficiency of using coordinates in developing tree competition indices is marginal when compared to coordinate-free methods.

2.4 Architecture and Operation of the Model System

The tree growth model system is composed of four main increment expressions for each species. These equations are used to estimate periodic changes in tree DBH, total height, and crown size. While crown size is not necessary to estimate volumes of individual trees, it is a primary predictor variable for DBH and height growth. As crown dimensions on trees change in response to stand density changes, consistent operation of the model system requires recognition of changing crown size during stand development. The fourth "increment" equation is a prediction of tree mortality.

The increment equations used in this model have been derived from growth plot data and consequently, their primary purpose is in modelling the increment of individual trees on plots. Foresters are generally familiar with procedures utilized to convert plot tree measurements into total plot volumes or volumes by log size estimates. The purpose of the increment equations is to provide some estimate of a tree-by-tree plot inventory record if the plot had been remeasured at some time in the future. Differences between successive plot inventory estimates provides an estimate of net plot growth. The predictions are for a time span of five years. Five years was chosen because it was the cycle length between measurements on most of the permanent plots available to this study. Predictions for multiples of five years are accomplished by a recursive process of repeated application of the increment models.

2.4.1 Input data requirements

Fifty year base age site indices are used as plot productivity indices. As there are site index differences between species, a site estimate must be available for each species. In the absence of field site measurements on all species present on any given plot, some general site conversions described by Watson et.al. (1979) can be substituted.

The tree information needed for model input is a list of the following items for each tree measured on a plot:

1) Species code
2) DBH to nearest tenth of an inch
3) Total height to nearest foot
4) Live crown ratio[2/]
5) Tree weight[3/] on a per acre basis

2.4.2 Model System Architecture

The increment equations are used to estimate changes in tree DBH squared[4/], total height, live crown ratio, and per acre weights. Mortality enters the system by changing the per acre weights associated with each tree. For a given species, these equations can be represented by the following implicit expressions

$$yd_{ijk} = f_d\{xd_{ijk}, \Theta d, Md_{ij}\}$$

$$yh_{ijk} = f_h\{xh_{ijk}, \Theta h, Mh_{ij}\}$$

$$yc_{ijk} = f_c\{xc_{ijk}, \Theta c, Mc_{ij}\}$$

2/ Live crown ratio is defined as ratio of the length of the tree bole covered with live branches to total height.

3/ The nominal geographical area used in modelling is one acre. For fixed area plots, tree weights are the reciprocal of the plot size. The weight is consequently an expansion factor for each tree in computing plot "per acre" statistics.

4/ The change in tree DBH squared rather than tree DBH was modelled because it is more directly related to tree volume.

$$yp_{ijk} = f_p\{xp_{ijk}, \theta p\}$$

where

ijk	are indices denoting the j^{th} tree on the i^{th} plot during the k^{th} five year growth period. (Subsequent indexing of expressions is deleted for the sake of conciseness.)
yd	Five year change in tree DBH squared, outside bark, in square inches.
yh	Five year change in total height in feet.
yc	Five year change in height to the base of the live crown in feet[5/].
yp	Probability the tree will die during the next five years.
f_d, f_h, f_c, f_p	Denote the implicit functions to predict yd, yh, yc, and yp respectively.
xd,xh,xc,xp	Vectors of explanatory variables used in predicting future tree increment. These variables are derived from the current tree list and include such things as tree crown dimensions, competition indices, and site index.
$\theta d, \theta h, \theta c, \theta p$	Vectors of model population parameters that are estimated from the data and determine the level of the increment estimate.

5/ In field determination of crown lengths, trees with asymmetrical crowns or "holes" in the foliage were visually reapportioned up the stem to get an approximation of an average complete crown length

Md,Mh,Mc Denote "equation modifiers" that are used to alter the predictions. (see below)

2.4.3 Estimates of the future tree list

Given the set of models previously described, each tree represented in the plot tree list has its characteristics updated by the following conventions to get an estimate of what it would look like if it was remeasured five years later.

Future tree DBH. If D_1 is the current tree DBH, an estimate of its DBH five years later (D_2) is given by

$$D_2 = \{D_1^2 + yd\}^{1/2}$$

Future total height. If HT_1 is the current total height, height five years later (HT_2) is estimated as

$$HT_2 = HT_1 + yh$$

Future crown ratios. If CR_1 is the current crown ratio, the crown ratio after five years (CR_2) is estimated by

$$CR_2 = \{(CR_1)(HT_1) - yc + yh\}/HT_2$$

Future per acre weights. If the current per acre weight is W_1, the weight five years later is estimated as

$$W_2 = W_1\{1. - (yp)\}$$

The updated tree list is then used to derive new vectors of explanatory variables (xd, xh, xc, and xp) for each tree and the prediction and updating process is repeated. Growth projections for any arbitrary multiple of five years can be accomplished by repeating this process. Hence, the growth model operates as a recursive system. Figure 1 shows conceptual changes in tree dimensions during one five year growth period.

2.4.4 Modifier Function

Equation modifiers are used to incorporate two different types of "random" factors into the model system. The first is considered the "calibration" factor. It is quite unlikely that the model system and parameter estimates developed in this study will exactly portray the growth of any tree or group of trees. Hence, when evidence is available to suggest that the system predicts low or high, this information can be used to adjust the system and produce more precise predictions. The methodology for accomplishing this will be described later.

A second type of "random" factor is incorporated to model "unexplained variation" in tree growth. It would be somewhat heroic to expect a simple system of equations to be capable of totally explaining the development of all trees in a complex biological system such as a forest stand over periods spanning several decades. There will be some variation that cannot be accounted for. Some preliminary analysis indicates somewhere between forty and seventy percent of the variation in sample data can be accounted for by a fitted model system. Hence, the unexplained portion is a substantial factor. For proper functioning of the system of models, the unexplained components of variation need to be explicitly recognized and incorporated. The reasoning for this may be clarified with the following example.

Consider a plot made up of saplings where all the trees are identical in terms of the characteristics incorporated in the models. Applying the increment equations to these trees would result in identical predictions for all trees. After a fifty year projection, all the trees would have the same predicted characteristics. However, we know

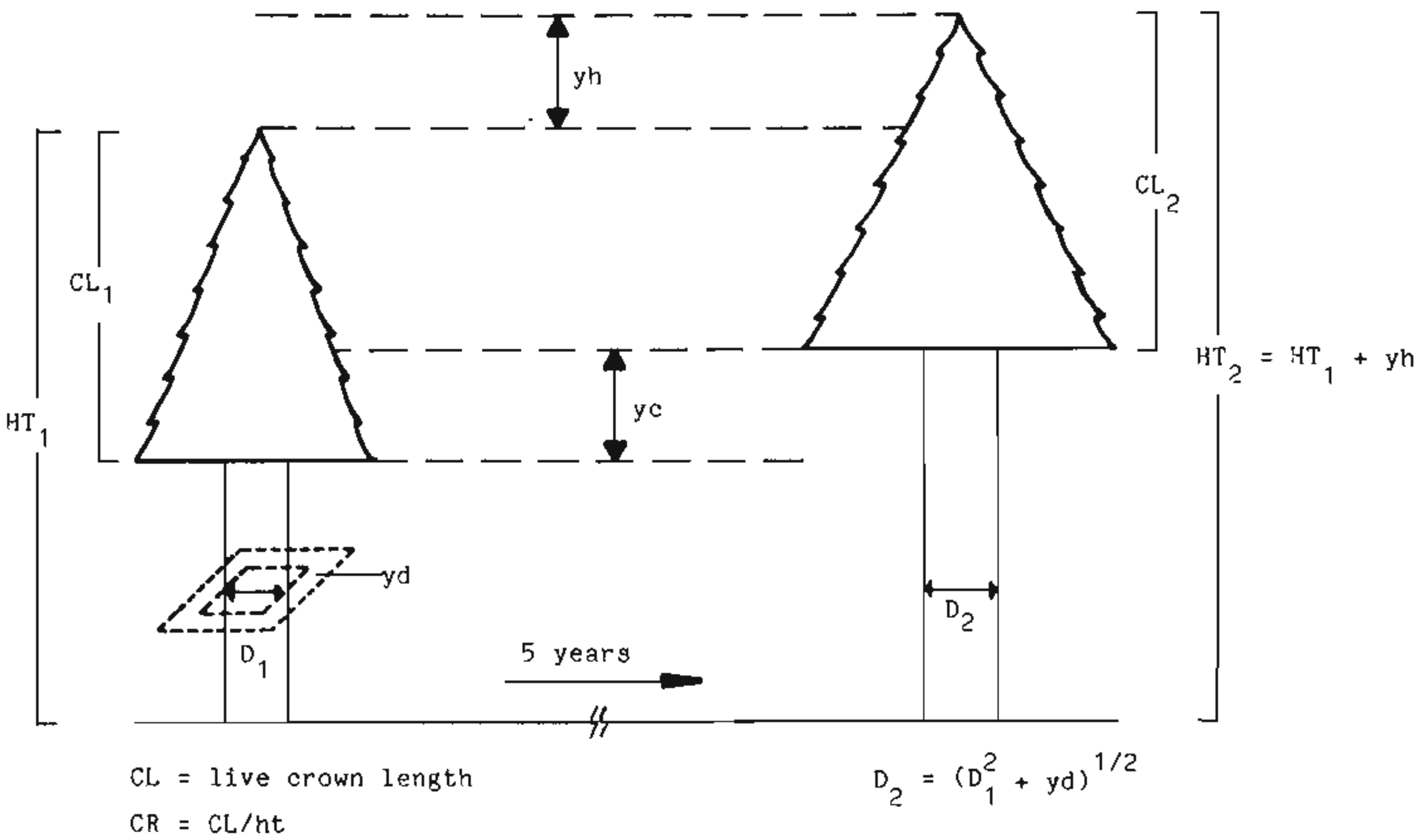

Figure 1. Conceptual changes in tree dimensions as perceived in modelling.

this doesn't happen in practice. Trees differentiate into different crown classes presumably because some trees grow slower or faster than others due to factors not specifically incorporated in the models. Also, competition subsequently acts to accelerate this differentiation.

In prediction, this problem results in unrealistic estimates of size class distributions among trees. Moreover, total plot volume estimates tend to be biased as the volume equations are not linear functions of tree size. The average value of a function is not the same as the function of the average value in this instance.

Random variation about a statistical mean in tree growth may be due to several sources. These might include catastrophic events such as logging damage, insect or pathogen related growth losses, region wide or localized periodic climatic fluctuations, genetic and microsite variability, and several others. While these sources may be significant and incorporated in the analytical phase to increase precision in parameter estimates, specific recognition of all factors tends to unduly complicate model building. Hence, it was felt that recognition of two general sources would be sufficient. These are a) time invariant random effects between trees (tree effects) and b) time variant effects among individual trees over time (periodic effects). In model operation, the distribution of tree effects in the population is considered to be the main source of variation to incorporate in equation "modifiers". Having some trees "grow" slower or faster than the model "norm" throughout an entire simulation induces size class differentiation that can mimic the the variation that is actually inherent in forest growth processes. Simulating additional random variation in the growth of individual trees over time, while being more realistic, was not considered to result in much difference in an overall growth simulation. Hence, tree effects were considered to be the main item in developing modifiers. The scheme used is explained in Chapter 6.

2.5 Resolution of Design with Objectives

Assuming the implementation of the model system is adequate, it can reasonably satisfy the objectives stated in Chapter 1. As the tree-by-tree analogs of the stand sample inventory data are used as input, the initial stand description used to start a growth projection is as specific as the sample data used to describe it. As a facsimile of raw inventory data is both an input and output of the model, it can be easily interfaced with other components of overall management information systems. Harvest simulation within the model system potentially has a high degree of flexibility since any combination of species and size classes can be "removed" from the tree list at any time during a projection. While the mechanics of calibration have not yet been described, explicitly recognizing this component in the design and analytical phase provides assurance of the compatibility of its eventual incorporation.

Chapter 3.

METHODOLOGICAL CONSIDERATIONS

This chapter delineates evaluation criteria that were used to guide the modelling process. A general analytical goal is framed in terms of an ideal quantitative procedure that could be followed to implement the model. Major anticipated problem areas stemming from data deficiencies and analytical misinterpretations are subsequently described.

3.1 Evaluation Criteria

Evaluation of any proposed or implemented model system may be approached from two different viewpoints. The first is operational utility: has the system been designed to encompass the range of plausible stand conditions for which growth projections might be desired; can simulation results be tailored to specific forecasting data requirements? The design characteristics described in Chapter 1 and 2 were incorporated based upon these considerations. A second means of evaluation is based on performance utility: how well do the projections match the growth of actual stands?

The type of situations where this model system has the greatest utility is in predicting the future development of existing forest stands. As these events have not yet taken place, direct evaluation isn't possible. However, there are three possible indirect ways of ensuring stability and overall accuracy of the implemented system: 1) choosing model forms that are in conformance with what is generally known about the growth and development of trees, 2) using estimation techniques that result in efficient and unbiased predictions, and 3) comparing projections of past stand development with what actually occurred under the assumption that a growth model system that is adequate in modelling historical stand development should be adequate in modelling future stand development. This latter aspect is the subject of Chapter 6.

3.2 Choice of Model Forms

Choice of a functional form for any of the component increment equations can be selected from a spectrum ranging between two separate viewpoints: (1) empirical positions with measures of statistical fit being used to determine the "best" form or (2) biologically-based models with some or all of the parameter values being derived from theoretical considerations.

Strict adherence to the former philosophy is felt to be undesirable as extrapolations beyond the data base used to fit the models would be an artifact of the analytical phase rather than an explicit consideration of how trees might perform in environments different than those used in a primary sample.

The latter philosophy, while being attractive, requires an acceptable tree growth theory that can be adapted to the operational objectives and level of resolution at which the model system is intended to operate. No completely adequate theories are known to exist. Operationally, items of interest are estimates of biologically somewhat arbitrary tree attributes (e.g., change in tree DBH squared). Hence, it would only be coincidental for a biological theory to produce functional model forms and some *a priori* estimates of parameter values applicable to the forest resource under study.

The modelling philosophy followed in this study is a compromise between these two extremes. Biological considerations are introduced by using tree crown size relationships as primary explanatory variables in quantifying tree bole growth. The branches of a tree contain its photosynthetic organs and therefore are directly related to a tree's potential ability to grow. Parametric forms incorporating crown dimensions are subsequently developed from preliminary screening of sample data and estimated by statistical methods. These procedures are described in Chapter 5.

3.3 Statistical Considerations in Model Implementation

Assuming that adequate functional forms for the model system have been selected, subsequent development can be treated as a problem is statistical design and estimation. From this viewpoint, there are three important features of the model system: 1) For a given tree at any time, the equation system is used to estimate three dimensional changes. Hence, the system is potentially a "simultaneous equation model" (Maddala, 1977); 2) The system is not only used to predict a time series of sequential increments on individual trees, but it is also used to predict growth differences between trees with different characteristics in different environments or cross-sectional phenomena. Hence, the system can also be viewed as a mixed time series and cross-sectional model. 3) If some variant of the open grown tree concept is employed in formulation, major variables available to develop a potential growth expression are tree height, DBH, and crown size. All of these variables are cumulative increments and represent special cases of distributed lagged forms of the variables we are attempting to predict. Hence, the overall model can be viewed as a recursive system.

Simultaneous consideration of these features is a source of complexity in analysis. Before any estimation procedures can be considered, model building requires a statistical characterization of the model system. This takes the form of a set of assumptions about which elements of the model system are variables and which are parameters, and plausible relationships among the random components of the system. A consideration of these assumptions are necessary in developing estimation procedures that are in some sense best. In the following two sections, a characterization of the model system is developed and a sketch of what is considered to be an ideal analytical procedure that could be followed to implement the model system is described.

3.3.1 Statistical Characterization of the Model System

If we initially consider a single tree, indexed as the j^{th} tree ($j = 1,2,3,\ldots J_i$) on the i^{th} plot, for which K_{ij} sequential observations are available, the following model can be postulated[1/]:

$$
\begin{aligned}
yd_{ijk} &= f_d(xd_{ijk},\Theta d_{ij}) + ud_{ijk} \\
yh_{ijk} &= f_h(xh_{ijk},\Theta h_{ij}) + uh_{ijk} \\
yc_{ijk} &= f_c(xc_{ijk},\Theta c_{ij}) + uc_{ijk}
\end{aligned}
\tag{3-1}
$$

The definitions here are the same as in Chapter 2, only the vectors of coefficients [Θd_{ij}, Θh_{ij}, ...] are considered to be specific for the ij^{th} tree and the u_{ijk} terms are random disturbances with assumed zero expectations associated with the k^{th} observation of tree j on the

---

1/ The mortality component is temporarily ignored in this section and the framework for discussion is "conditional on a tree remaining alive".

i^{th} plot.

The following notation is introduced to facilitate referring to sets of elements in equation system. We define

$$\theta_{ij} = [\theta d_{ij} | \theta h_{ij} | \theta c_{ij}]$$
$$x_{ijk} = [xd_{ijk} | xh_{ijk} | xc_{ijk}]$$
$$y_{ijk} = [yd_{ijk} | yh_{ijk} | yc_{ijk}]$$
$$\upsilon_{ijk} = [ud_{ijk} | uh_{ijk} | uc_{ijk}]$$

To recognize that the current values of the elements of x_{ijk} are based on past values of these variables and interim growth (tree size, for example), a recursive feature of the model can be implicitly represented as

$$x_{ijk+1} = g_r(x_{ijk} , y_{ijk} \quad j = 1,2,...J_i)$$

In practice, θ_{ij} will seldom be known and highly impractical to estimate for every tree for which projections are required. One practical way to accomplish a characterization is to consider the individual tree model coefficient vectors as random and treat the model (3-1) as a random coefficient regression (RCR) model in analysis (cf. Swamy, 1970). In a this model we let

$$\theta_{ij} = \theta + q_{ij} \tag{3-2}$$

where θ is the mean population coefficient vector and q_{ij} is a vector of the random differences $[\theta_{ij} - \theta]$. We can stipulate that

$$E(q_{ij}) = 0$$

To recognize that there may be correlations among some trees on plots (due to, say, genetically similar origins, microclimates, etc.), we can represent q_{ij} as

$$q_{ij} = \lambda_i + \delta_{ij}$$

where (analogous to a nested random effects model (Scheffe, 1959)), λ_i is a vector denoting the "effect" of the i^{th} plot and δ_{ij} is the "effect" of the j^{th} tree on the i^{th} plot. Plausible assumptions are to consider the sets $\{\lambda_i\}, \{\delta_{ij}\}$ completely independent with elements being identically distributed and having variance-covariance matrices Σ_λ and Σ_δ respectively. These assumptions can be represented by

$$
\begin{aligned}
E[\theta_{ij}-\theta)(\theta_{mn}-\theta)'] &= \Sigma_\lambda + \Sigma_\delta && \text{for } ij = mn \\
&= \Sigma_\lambda && \text{for } i = m,\ j \neq n \\
&= 0 && \text{otherwise}
\end{aligned}
$$

The relationships stated above should be sufficient to implement the model system. An estimate of the mean population coefficient vector θ can be employed to make "mean" predictions for any tree. The λ_i and δ_{ij} operationally represent "time invariant" random effects which are to be incorporated into the model as deviations from θ. While, in general, not being known or estimable for projection purposes, if some parametric form for their distributions are assumed (e.g, multivariate normality), then the estimates of Σ_λ and Σ_δ can be used to simulate stochastic effects in model operation.

3.3.2 Proposed Analytical Procedures

The major estimation objective is to produce estimates of the mean population coefficient vector θ and the variance-covariance matrices of the individual tree parameter vectors (Σ_{λ} and Σ_{δ}) that are in some sense "best". For example, a common goal in statistical analysis is to seek unbiased minimum variance estimates of parameters, or when this is unobtainable, the asymptotic properties of consistency and efficiency are accepted instead.

In the initial phase of estimation, a major focal point is in estimating the parameter vector of a single tree (θ_{ij}) and its sampling variance, $V(\theta_{ij})$. Development of estimates that are in some sense best would at least require specifications and estimates of the variances and covariances of the error terms $\hat{V}(u_{ij})$), not only over time but also across equations. Assume that such problems are solvable and analysis produces the the best estimates possible ($\hat{\theta}_{ij}$, $\hat{V}(\theta_{ij})$, $\hat{V}(u_{ij})$).

Model building requires the first phase to be repeated for several trees in a random sample. An important characteristic of the random sample is that all trees that have obtained some minimal size are equally likely to be sampled. This requirement will be discussed in more detail later.

From the sample-based sets $\{\hat{\theta}_{ij}\}$, $\{\hat{V}(\theta_{ij})\}$, $\{\hat{V}(u_{ij})\}$, the second phase of model building is to estimate θ, Σ_{λ}, and and Σ_{δ}. Swamy (1970), for instance, provides a general theoretical overview of estimation and hypothesis testing in a single equation linear RCR model which combines both time series and cross-sectional observations. Extension of this work to multiple equation systems, and possibly nonlinear models, does not appear to be theoretically unmanageable (cf. Bard 1974) and could be adapted here for our purposes.

One final analytical phase would be a simplifying one aimed at finding a partition of Θ, say $[\Theta1|\Theta2]$, such that $E[(\Theta1_{ij} - \Theta1)(\Theta1_{ij} - \Theta1)']$ can effectively be considered a null matrix. In other words, some coefficients can be considered the same for all trees and would indicate that the impact of certain factors influencing tree growth is similar (zero variation) for all trees in the population. From a practical standpoint, this is desirable because it would reduce the complexity of a stochastic scheme.

The preceding sketch is considered to be a reasonable analytical framework to guide subsequent implementation of the model system. Treating interindividual differences in tree growth in the context of a RCR model provides a general, flexible, and analytically translatable means of accounting for heterogeneity in growth between trees in a population. Relevant parameters are identified and serve as a focal point for subsequent estimation.

As an analytical goal however, the above development has to be considered an unobtainable ideal. Numerous difficulties, stemming largely from data deficiencies and sampling problems prevent a straightforward analytical adaptation. Nonetheless, where less than optimal or *ad hoc* procedures are pragmatically necessary, the analytical ideal can be used as a basis in judging the adequacy of any proposed procedure.

3.3.3 Empirical Support for the Statistical Characterization

One immediate criticism that can be raised, particularly if the proposed characterization of the tree growth process is difficult to implement, requires some justification that there is sufficient "time-invariant" effects among the population in question to warrant consideration of the proposed approach. While there is no apparent published evidence explicitly in the context of tree-based increment models, there is sufficient indirect evidence to indicate that these sources of variation are general attributes of tree populations.

Mitchell (1975) performed stem analysis on several plots in young Douglas fir plantations where height growth competition was purportedly negligible. Growth on individual trees was smoothed over time (presumably removing time variant tree effects) and the standard deviation of current height growth of trees within plots was estimated to be between 15 and 20%. Krumland and Wensel (1977b) found the standard error of estimated site index among dominant and codominant trees on individual plots to be about 7%. Since the sample was restricted to the "better" trees on plots, the results are not inconsistent with Mitchell's.

Differentials in growth potential between trees is also an apparent prime justification for geneticists attempting to develop faster growing strains of trees. Staebler (1972) estimates that increases in volume growth of 25% are reasonable figures to expect from first generation tree breeding programs.

If, after an analysis, residuals from a "mean" population growth model for several observations on the same tree were found to be highly correlated, it would support the idea that much of the unexplained variation about the fitted model could be due to "time invariant" tree effects (some trees consistently grow faster or slower then the population average). Dale (1978) correlated sequential residuals of tree predictions from a model fitted by conventional analysis and estimated a correlation coefficient r = .89. Curtis (1975) paired basal area growth residuals from an analysis based on total plot growth and reported an r^2

= 0.69.

Implications of this type of evidence in terms of model interpretability have been largely ignored by other researchers. However, they are of sufficient magnitude to support the previously described characterization.

3.4 Statistical Problems in Growth Model Development

The major problem in developing the model system derives from not having time series data of sufficient length to implement the procedures described in section 3.3. The bulk of data available for modelling consists of single growth measurements on a large number of individual trees of each species. Almost all documented tree models are based on similar types of data due to the general scarcity of growth observation sequences of sufficient length to allow growth models to be constructed for individual trees. In the absence of time series data, a common analytical procedure (e.g. Stage,1973; Mitchell,1975) has been to to use the same functional form that is used for an individual tree growth model but to perform an analysis that is entirely cross-sectional; that is, the coefficients for one model are estimated using single growth observations from several trees. This type of analysis is sufficiently different from what was previously described to warrant a formal distinction between the two types of models:

Model Type A. Referring to section 3.3, growth models would be fit separately to individual trees based on time series data. The sets of coefficient estimates obtained from these procedures would then be averaged to produce an estimated mean coefficient vector. Predictions from the resulting model can be interpreted as describing the growth trajectory of an average tree in the population.

Model Type B. This type of model form would result from ignoring the time series attributes of the data, out of choice if time series data were available or out of necessity if the data were composed of single growth measurements on individual trees. All available data are pooled and the same functional model form is fit as was used for individual trees in a type A model. This directly produces one vector of estimated coefficients. This procedure is analogous to an ordinary regression analysis to predict growth. The conventional interpretation given to a regression model is that it predicts the conditional mean response of the sample given specific values of the explanatory variables.

In the absence of time series data, it is tempting to conjecture that the estimates obtained with a type B analysis could be used as the mean coefficient vector in implementing a model for which type A attributes are desirable. In otherwords, we perform an analysis based wholly on cross-sectional data (single growth measurements on several trees) and assume the resulting coefficient estimates are applicable in predicting a growth time series for a single tree. There are four factors however, which, in combination, make such an assumption statistically untenable:

1) Tree growth is expressed, in part, as a function of tree size.
2) The model system, as it is intended to be used, operates in a recursive manner. Predictions of current tree growth are added to current tree size which is then used to make future predictions of tree growth. As a consequence, the predicting variable (tree size) as well as tree growth are both random variables in the analysis.

3) There is presumably sufficient time-invariant variation in individual tree growth to warrant considering certain coefficients in a growth model are not the same for all trees.

4) In the common case, lack of time series data is the result of only limited experimental control being exercised in the collection of tree growth data. For practical purposes, single growth measurements on individual trees have to be selected from existing stand conditions, and necessarily, existing tree size distributions.

The following example is provided to illustrate the interpretive differences between type A and type B models.

Suppose we have randomly selected "J" trees, all which have obtained some minimum age, and we have subsequently observed their growth from ages 1 to "T" years. If tree growth is considered a continuous process, the following model can be posed for the j^{th} tree.

$$\delta y_{jt}/\delta t = A_j - by_{jt} \qquad t = 1,2,\ldots T \tag{3-3}$$

where

$\delta y_{jt}/\delta t$ = 'instantaneous' growth rate at time t for tree j

y_{jt} = size at time t

b, A_j = growth model paramaters

The growth model (3-3) is a simple linear function of tree size. In this formulation, we assume that the coefficient "b" is invariant between trees but A_j may vary from tree-to-tree. To characterize the entire population of trees, we assume that A_j is related to the population mean coefficient A by

$$A_j = A + a_j$$

and a_j is considered to be a random tree effect having an expectation of 0 and a variance of σ_a^2. For this example, time-related stochastic disturbances are assumed to be nonexistent. If tree size at time 0 is also assumed to be 0, the integral of (3-3) is the well known "monomolecular" growth function (Grosenbaugh, 1965).

$$y_{jt} = (A_j/b)(1 - e^{-bt}) \tag{3-4}$$

where t = tree age and A_j/b is the maximum size tree "j" will attain. Figure 2 shows a family of curves generated by equation (3-4) delineated by a minimum and maximum sampling age. This is the conceptual sampling frame for tree size. Figure 3 is a translation of this sampling frame showing the growth rate $\delta y/\delta t$ as a function of size, y.

This figure delineates a bivariate sample space as both current growth and tree size are random variables. While the distribution of tree ages in the coastal forest resource is unknown, it is useful for this example to assume a uniform distribution. This would correspond to the classical balanced even-aged forest with the exception that mortality is ignored here. If a_j is assumed to be normally distributed, figure 4 graphically depicts the joint density of $\delta y/\delta t$ and y. There are two lines shown. Line "A" which corresponds to the mean population growth relationship ($\delta y/\delta t = A - by$) and is what would be produced by a type A model analysis. The lines parallel to line A can viewed as anologs of growth models for individual trees which, when averaged, produce line A. Line "B" has been drawn to approximate the conditional mean growth given tree size (y). This line is the analog of what would result from a type B analysis where one model is estimated with data pooled from all trees.

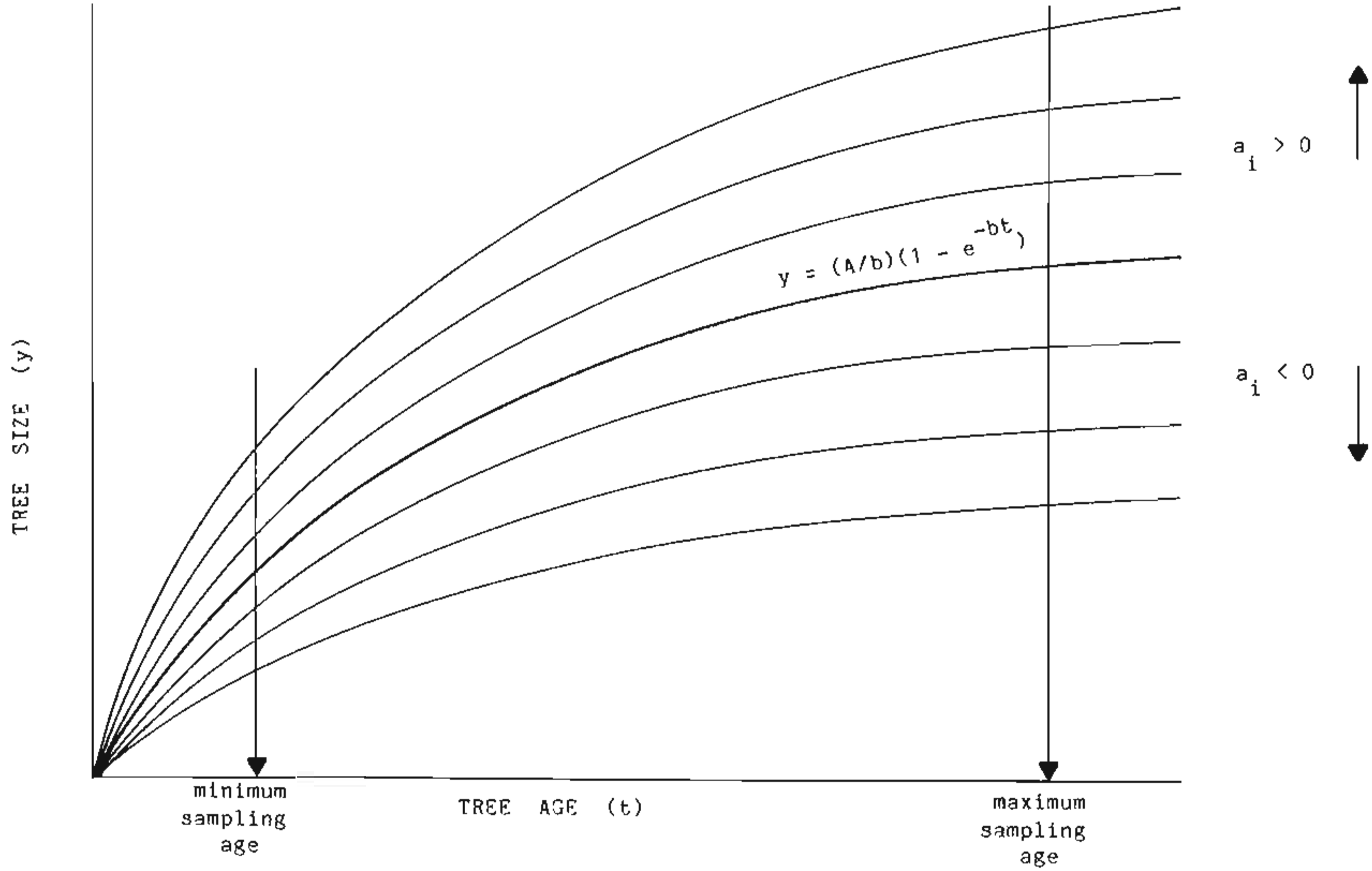

Figure 2. Family of cummulative growth curves generated by the function $y = (A_i/b)(1 - e^{-bt})$

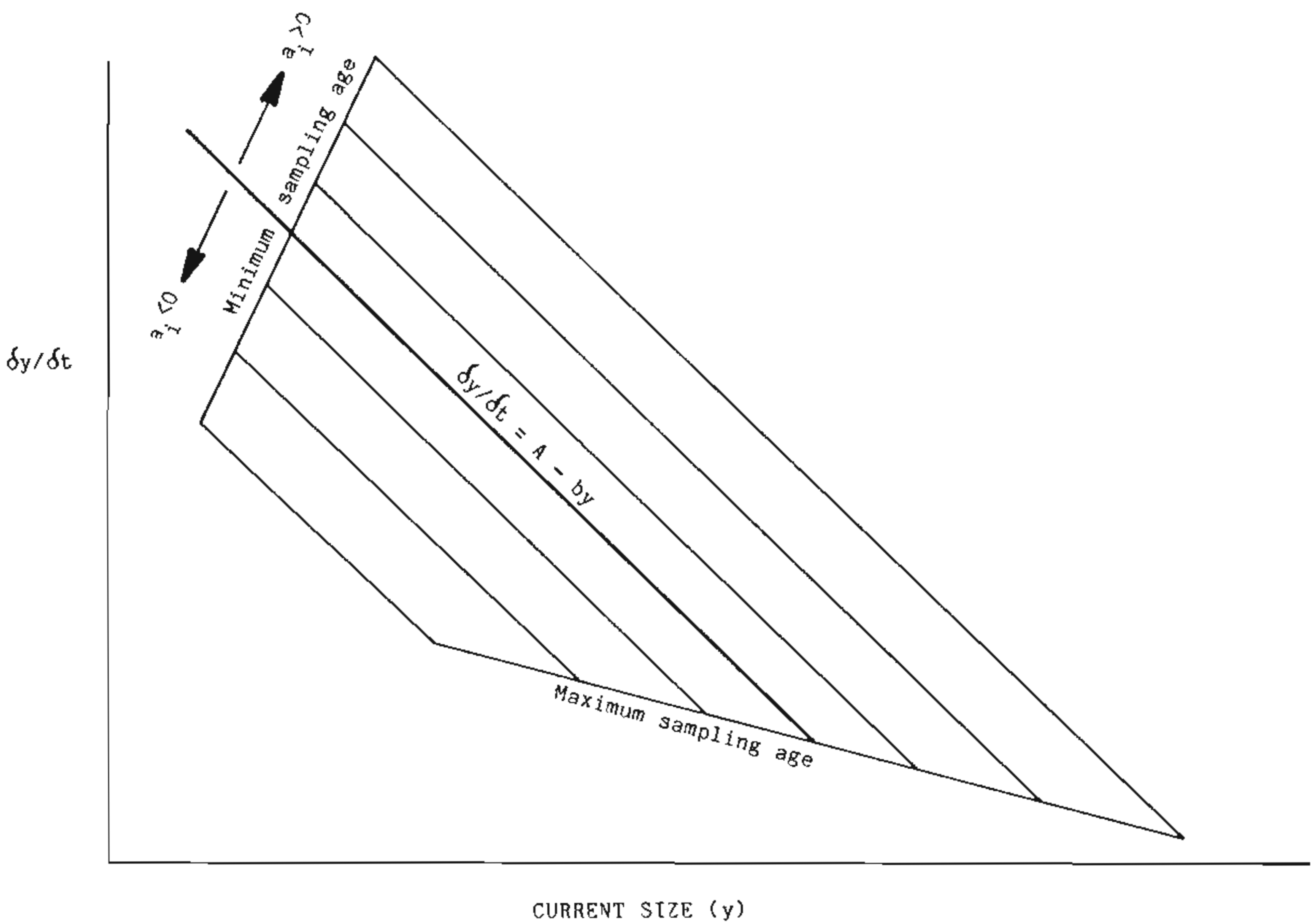

Figure 3. Family of growth rates expressed as a function of current size.

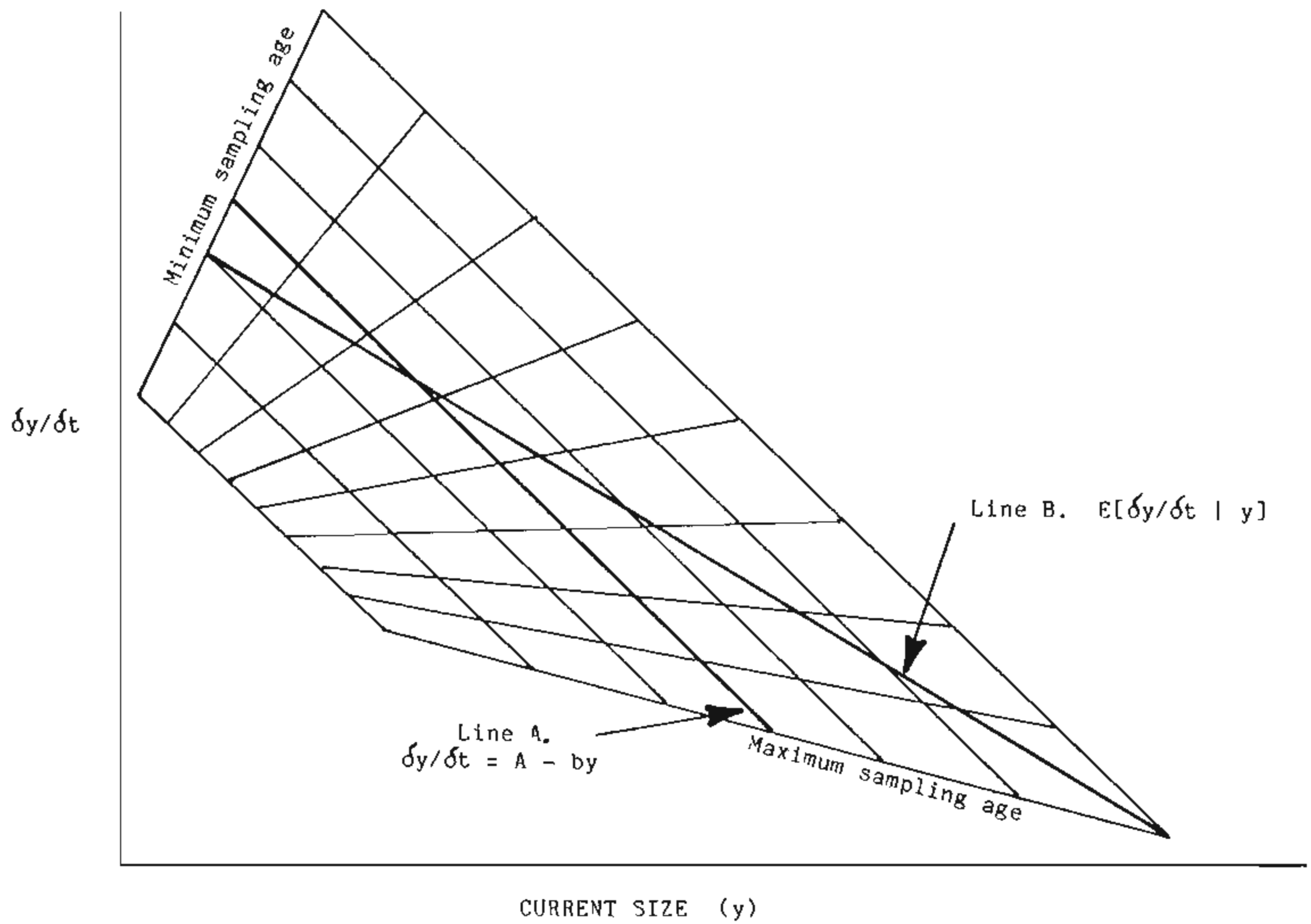

Figure 4. Joint distribution of current growth rates and current size.

The difference in prediction between the two models results from the distribution of random tree effects (the a_j) not being independent of tree size. Trees with $a_j < 0$ are relatively slow growing and never get very large. Consequently, there are more "slow growing" small trees than fast growing small trees which results in the average value of a_j being less than zero for small trees. The converse is true for the larger trees in the population. That is, larger trees are larger because, in part, they grow faster.

The major consequence of this phenomenon is that if a type B model is developed from the data, the coefficients are biased if interpreted in a type A model context (i.e. the model is used to recursively predict the growth of the "average" tree over time). The tree effects (a_j) are omitted from the model and are subsequently reflected as an error term. If the coefficients in the type B model are estimated by least squares, a necessary condition for the coefficients to be unbiased is the requirement that the error terms in the model (which are the a_j) be independent of the explanatory variables or tree size (cf. Maddala, 1977, p.448). By the argument previously given, this condition is violated. The predictions resulting from a type B analysis consequently reflect the "true" type A tree growth relationship <u>plus</u> an additional effect representing the conditional mean tree effect for a given tree size.

Usually in natural populations of trees, there is some decrease with age in the numbers of trees due to mortality. In even-aged stands of trees, for example, it is not uncommon to have half of the trees die between the ages of 20 and 100 (e.g. Lindquist and Palley, 1967). To the extent that much of the mortality is concentrated in the smaller poorer growing trees in a stand, the actual divergence between lines "A" and "B" is underestimated.

3.5 Interpretative Differences Between Model Types

All regression analyses can be thought of as answering questions and what seems relevant here is what questions do each of these model types answer? If we are interested in answering the question of not only how growth varies between trees of different sizes in our population but also how individual trees grow over time, then using a type A model and analysis is appropriate and is also in conformance with our usual expectations about tree growth models when used to recursively make forecasts for more than one time period.

If we are only interested in estimating average current growth rates in a population of trees similar to those used to develop the model, then a type B model should be used. In this type of modelling context, least squares estimates are both unbiased and efficient because tree size is no longer considered to be a random variable. Hence, the previous problem with the errors in the model being correlated with the explanatory variable is no longer a concern.

While a formal mathematical development is omitted, it should be fairly obvious that if if there is no variation in tree effects in the population, ($\sigma_a^2 = 0$), then both types of model analyses will produce comparable analytical results which can be used interchangeably in a type A or a type B model application.

Startup Problems

Assuming we have estimated the mean coefficient vector in a type A model, there is a "startup" problem in predicting the growth of stands that already exist (i.e., the stand is composed of trees greater than the conceptual minimal size). While we can assume that the tree effects have zero expectations in the population as a whole, this assumption will probably not be valid for the "average" tree in a specific stand with a known size distribution. This is essentially the operational situation faced when a tree list for a specific stand is used to initiate a simulation. Relative to the previous example, if the stand is composed mainly of large trees, the mean value of a_j for this stand will be greater than zero. Conceptually however, the average difference in predictions between the two model types applied to trees in the list provides an estimate of the conditional mean a_j given a specific stand. A variant of this concept can be used to develop general "start-up" calibration functions to align the level of predictions of models in the system at the start of a simulation.

3.6 Implementation Problem Summary

The way the growth projection system is intended to operate implies the implementation of a model type A, particularly if time-invariant tree effects are a significant source of variation in tree growth. The major difficulties in implementing the system for the coastal forest resource stems from the lack of adequate time series data on individual trees and intrinsic selection biases resulting from sampling existing stands of trees. These problems are, in general, common to modeling efforts of this type and do not seem to be easily remedied in the near future. As a consequence, development of *ad hoc* procedures in an attempt to approximate results that might be obtained from application to the analytical methods proposed in section 3.3 are necessary and justifiable for pragmatic reasons. Results and performance of the procedures applied in this analysis should be valuable to other researchers attempting to develop improved growth models with similar types of data.

Chapter 4.

MEASURES OF COMPETITION

It is generally recognized that individual trees in dense stands grow less in dimensions than their counterparts in more open stands. Similarly, in a given stand, understory trees tend to grow less than overstory trees. Both of these observations allude to the more general phenomena of intertree competition.

Distance dependent tree modelers have proposed numerous indices of competitive stress for individual trees. Alemdag(1978) and Johnson (1970) have evaluated several of these measures. These indices take into account, directly or indirectly, the number of competitors, the size of the subject tree and each competitor, and the distance between the subject tree and each competitor. All of these methods potentially exploit a three dimensional coordinate system (two planar coordinates and tree heights).

In distance independent tree modeling, planar coordinates are unavailable. Consequently, competition effects are usually incorporated in distance independent tree models as some function of a measure of stand density and relative size, usually the ratio of tree diameter to average stand diameter (Stage 1973, Dale 1978, Goldsmith 1976). Hann (1980), in a diameter class model, used basal area in the two adjoining diameter classes, basal area in smaller classes, and basal area in larger classes for competition measures.

While the density-relative size approach may, in general, provide adequate measures of competition, there are plausible stand treatments where the approach may lead to insensitive or inconsistent predictions. For example, density measures are usually negatively correlated with tree growth while relative size measures are positively correlated. A thinning from below reduces stand density and raises the average stand diameter. If relative size is defined as the ratio of tree DBH to average stand DBH, then the post-thinning relative size of most trees will decrease from pre-thinning levels. From a biological viewpoint, any form of harvesting reduces, or at least does not increase competition effects among the remaining trees. However, due to the way a model might be constructed, the induced decrease in relative size may not be offset by a decrease in density and it may be possible for the post thinning prediction of tree growth to be less than the pre-thinning prediction. Also, the ratio of tree DBH to average stand DBH can be interpreted in two different ways: (1) as an index of competitive stress in which case it is a causal factor in tree growth processes and/or (2) as a measure of random tree effects in which case it can be viewed an outcome or effect of the forest growth process rather than a causal factor.

In mixed species stands, another complicating aspect of the density-relative size approach (particularly when DBH is used as the size variable) relates to size differentials among species. In the simplest application of this approach, it is implicitly assumed that a tree of a given DBH contributes the same amount to the density or relative size index regardless of species. In typical redwood-Douglas fir stand mixes, it is not uncommon for the dominant redwoods to be larger in DBH than dominant fir but shorter in total height. Consequently, without considerable refinement, treating both species the same may lead to inconsistent predictions.

In view of these possible inconsistencies and ambiguities, density-relative size measures were not used and an alternative approach was developed instead.

Canopy cover percent is a familiar concept to foresters, particularly in remote sensing applications. It is frequently expressed as the proportion of the ground area occupied by the vertical projection of tree crowns, that is, canopy cover at ground level. In a more general sense, if we sum the crown area for all trees, it is quite possible for the canopy cover percentage to be greater than 100% because the crowns overlap. If we begin to take "horizontal slices" through the stand at different heights, the canopy cover percent will decrease until at the tip of the tallest tree, it is zero. If canopy cover percent is expressed as a function of height above ground, different stand structures will display different "canopy cover profiles". Figure 5 shows representative profiles for even-aged, all-aged, and two storied stands.

Intuitively then, the canopy profile can provide an index of density at different heights on a given plot. It can be thought of as being related to or reflecting average light availability at a given height above the ground and as such, provides some measure of competition. Before developing an explicit competition measure, a description of the method used to quantify the canopy cover profile is in order.

4.1. Computation of Canopy Cover Profile

The basic information available for modeling the canopy cover profile consists of total height (HT) and live crown ratio (CR) of each tree on a plot. From these two variables, we adopt the following conventions to compute crown length (CL) and height to the crown base (HCB):

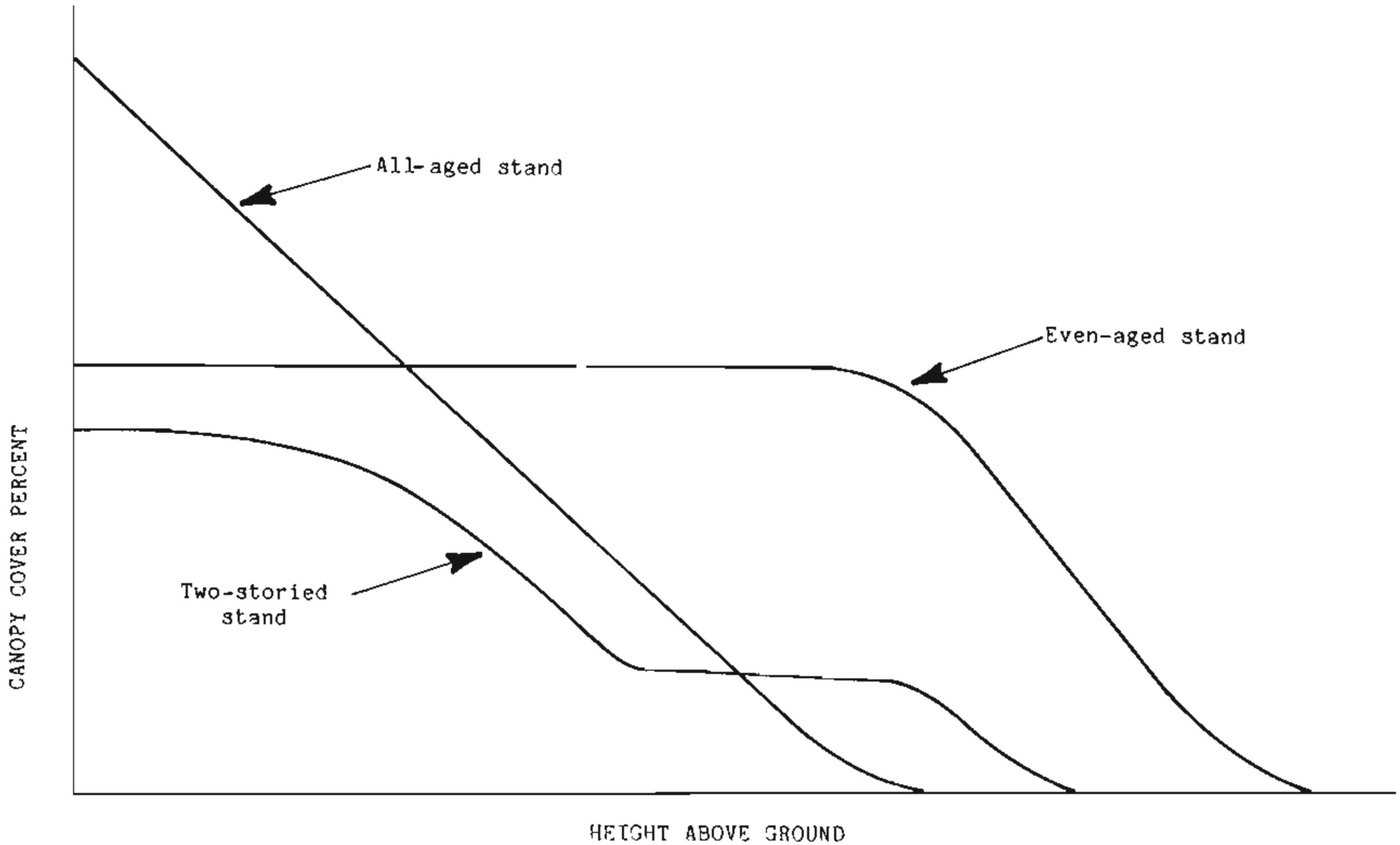

Figure 5. Illustrative canopy cover profiles for stands of different structure.

$CL = (HT)(CR)$

$HCB = HT(1 - CR)$

To use this information to develop a crown canopy profile, crown radii at different points throughout the entire live crown on each tree is needed. As this type of information is difficult and expensive to collect, a model was used to estimate crown radii. Casual inspection of forest grown conifers indicates that the crown profile of individual trees is somewhat parabolic above the zone where crowns of adjacent trees begin to overlap. Below this point, the profile is somewhat cylindrical because branch growth is retarded due to poor light conditions and possibly mechanical effects due to branch interlocking. The lowermost branches may even be shorter than higher branches.

Mitchell (1975) developed a crown width model for Douglas fir in the Pacific Northwest. Using coefficients he provides, the following approximation can be obtained:

$$CW_i = 22.503\ (\ln\ (L_i/20 + 1)) + d_i \qquad (4\text{-}1)$$

where

L_i = Distance in feet from tree tip to a point "i" in the tree crown

$\ln(x)$ = natural logarithm of "x"

d_i = tree bole diameter in feet at point i

CW_i = crown width in feet at point "i".

This expression is only for the portion of the tree crown above the general zone of branch contact with other trees.

Insufficient data were available to estimate the coefficients in equation (4-1) for each of the species groups we are modeling, however, a spot check with a small amount of data indicated that equation (4-1) provided a fair approximation for young-growth conifers in the North Coast although there is considerable variation between trees. Since the basic objective here is to develop a consistently applied index rather than an absolute measure, equation (4-1) was used as a basis for developing a crown canopy profile. The following conventions and procedure were used:

a) The "d_i" term in equation (4-1) was omitted. This introduces a slight consistent underestimate. However, as the canopy cover profile is used only as an index, this was not considered to be a significant problem.

b) Equation (4-1) with d_i set to 0 was applied to all eight species groups.

c) The equation was applied to the entire crown of each tree with no adjustment for possible departures below the point of branch contact.

d) The canopy cover profile takes the form of a vector with consecutive elements representing canopy cover percent at 10 foot increments above the ground.

e) To provide estimates of this vector, equation (4-1) was applied to each tree to estimate crown widths at 10 foot intervals. Each of these crown widths was used to estimate crown area by assuming cross sections were round. Multiplying the areas by the tree's per acre weight divided by 43560 provides an estimate of the tree's contribution to the crown canopy vector. Below the base of the crown, the contribution was assumed to be the same as at the base of the crown. Figure 6 is a graphical representation of how the canopy cover profile is derived.

The weakest aspect in using the canopy vector as a density measure stems from assuming that hardwood crowns are exactly like conifer crowns in contributing to tree competition. Profile dimensions and light penetration qualities are noticeably different. To test whether this has a significant effect, a hardwood canopy profile was also computed for each sample plot used to develop tree increment equations and analyzed in the modeling process. Results were inconclusive due to the small number of sample plots that had significant numbers of both conifers and hardwoods. A more conclusive analysis will have to wait until better data sources become available.

4.2. Development of a Tree Competition Index

The crown size of a tree is directly related to its growth capabilities and the degree to which it is shaded would be a measure of how much its growth would fall short of the potential growth it could attain in an open grown or full sunlight condition. An initial thought was to use the estimated canopy cover percent at a point, say, in mid-crown of each tree as a measure of competition.

However, using mid-crown as a reference point would presume that for two trees of the same height on an individual plot, the one with the shorter crown would be assigned a lower competition measure. In undisturbed stands, trees with relatively long crowns tend to be ones adjacent to openings in the canopy and are in a more lightly stocked

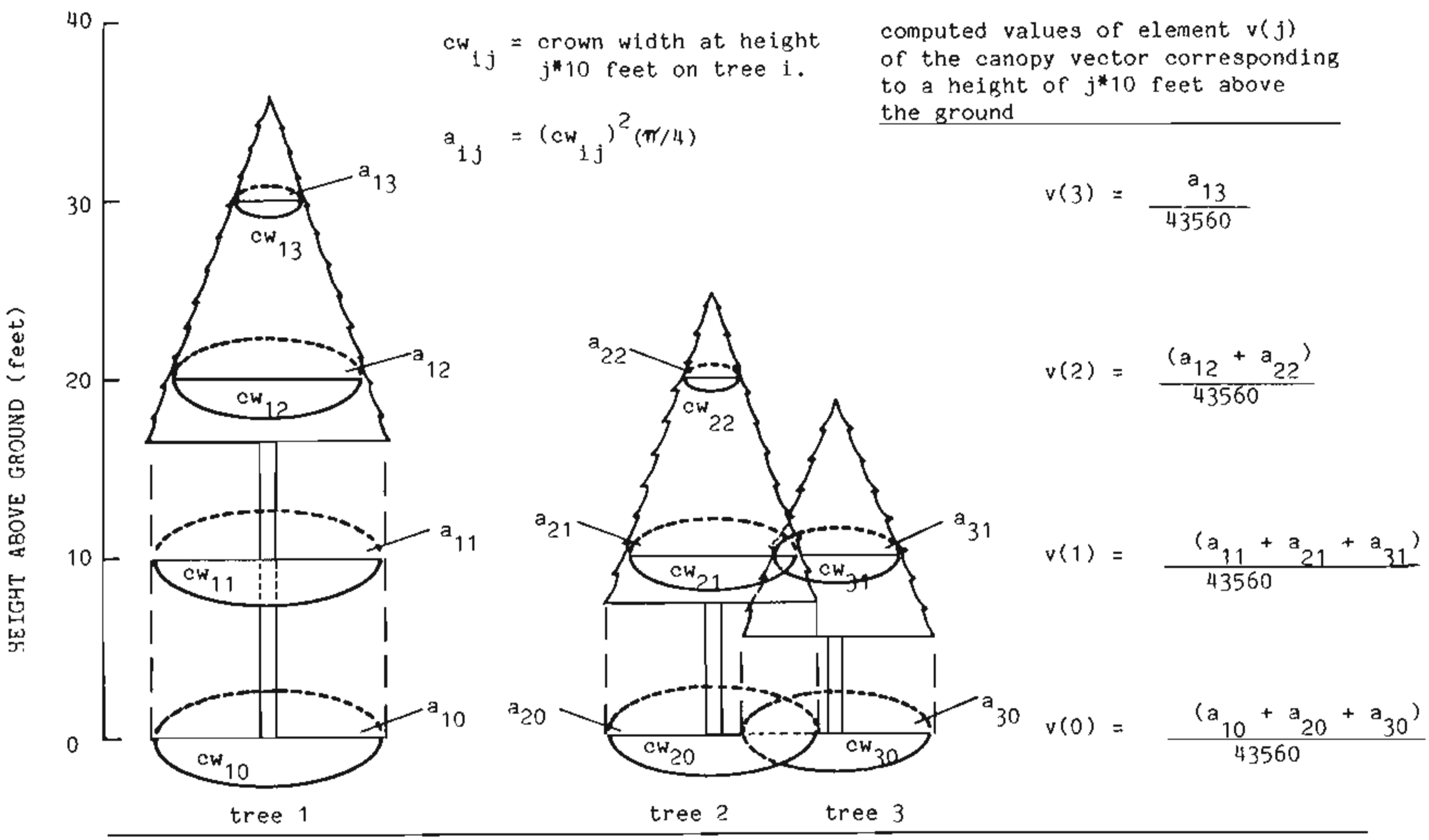

Figure 6. Conceptual example of how the canopy cover vector is computed for a one acre plot with three trees.

position within the plot. Conversely, trees with shorter crowns tend to be in relatively dense positions. This apparent anomaly in the relationship between crown length and canopy density stems from not recognizing spatial arrangements of trees.

As a compromise, reference points independent of crown length were chosen. These points are at some proportionate amount of total tree height. This assigns trees of the same height within a plot the same competitive index. Explicit forms of the competition index are detailed in the next chapter.

Chapter 5

ANALYSIS AND RESULTS

5.1 Introduction

This chapter provides a description of the data used in the analytical phase of this study and outlines the methods used in developing general functional forms for the structural components of the increment equations. Estimation procedures and methods for selecting which model coefficients might reasonably be considered random in the population as a whole are subsequently described. Model forms and the results of fitting equations to data are presented and discussed. Subsequently, the necessary relationships for the "modifier" components of the model system are derived and relevant parameters estimated.

5.2 Data Sources

The data used to derive estimates of the model system parameters used in this study have been drawn from an extensive collection of permanent and temporary growth plots located in Mendocino, Humboldt, and Del Norte counties. All of these plots are within the redwood-Douglas fir forest type and situated in stands of predominantly young-growth timber. Approximately 15% of the sample plots had one or more residual old-growth tree and 25% of the available plots had been subjected to some form of partial harvest prior to growth measurements. About 70% of the plots were from the coastal zone that is subject to fog influence and the remainder from the somewhat drier interior. Plots that were in the transition zone to the mixed conifer forest type were excluded from analysis. With the exception of Jackson State Forest in Mendocino County, all plots were located on private forest land. Most of the growth information had been collected during the calendar years 1952-1979.

Plots were all fixed-area plots ranging in size from one-tenth to one-half acre. For approximately two thirds of the plots, subplots were included for the measurement of smaller trees (less than 11.0 inches DBH).

Altogether, 412 plots were considered to be usable in one form or another for model development. These plots were screened from a much larger set with rejections predominantly based on the following items:

a) Data collection procedures did not give adequate measurements on individual trees. (e.g., no total heights or crown ratios were measured on any tree on a plot).

b) Collection procedures on individual plots were incompatible between measurements. (e.g., subplot sizes and/or minimum DBH recorded on the plots were changed between measurements).

c) Plots were not located in stand conditions generally representative of the coastal forest type or were otherwise of limited analytical use (e.g., highways or landings were located within plot boundaries; plots had been purposely located in unusually exceptional stand locations; plots were located in swamps, between cover type boundaries, or in situations where the treatment history was not uniform throughout the plot; plots were located in stands with exceptional amounts of wind throw, animal, or logging damage; landslides had occurred within the plots, or in general, the plot was not representative of a stand condition foresters would consider managing (e.g, "pygmy" forest land)).

d) Minimum DBH's recorded on the plots were not considered to be low enough to adequately represent the within-plot stocking. This was frequently a problem in sapling and poletimber stands.

The available permanent sample plots had been measured for growth one to four times. Thus, while providing some time series information, the number of sequential measurements on individual trees was not sufficient for explicit time series analysis.

As the use of canopy profiles for the development of competition indices requires heights and crown sizes for each tree, these items had to be estimated for each tree lacking these measurements. Appendix I describes these estimation procedures and other data adjustments required for compatibility purposes.

Table 1 shows the number of sample trees by species that had some form of usable growth measurements. Subsequent analysis has made use of several subsets of the main data base. These subsets are described in more detail in later sections.

Table 1. Numbers of available sample trees by species with at least one usable growth measurement.

Redwood	Douglas fir	Tanoak	Alder
8168	2280	626	26

5.3 General Estimation Objectives

The primary objective in the initial stage of analysis is to develop models with type A attributes (cf. section 3.5). Specific emphasis is directed at obtaining reasonable estimates of the mean vector of coefficients for the three main structural growth models and to determine a suitable partition of the coefficient vectors between fixed and random components.

Simultaneous estimation of the coefficients in all three increment equations was not attempted because trees with all three increment variables measured were confined to only a small portion of the data base. To restrict the model development to this subset was not felt to be representative of the population as a whole.

The common denominator in the data set is a single growth measurement on each individual tree. Thus the data are only directly amenable to estimating type B models when what is needed is a model of type A. Consequently, an indirect procedure was used in analysis in an attempt to develop models with type A attributes and also to provide some indication of the variability in model coefficients between trees.

5.3.1 Data Stratification Methods

Unbiased estimation of the mean population coefficient vector for each of the component equations in the growth model system could be accomplished with single growth measurements if tree effects (deviations of individual tree model coefficients from the population mean) were known. Treating them as unknowns results in more coefficients than observations and prevents one from obtaining unique estimates. Ignoring the possibility of tree effects is presumed to result in biased coefficient estimates. However, if the sample could be stratified into classes representing similar tree effects, then each class could be considered a "pseudo-time series." Ideally, the within-stratum variation in tree effects is negligible and the potential biases in the within-stratum model coefficient estimates resulting from tree effects being correlated with the explanatory variables is reduced. Coefficient estimates for each stratum could then be combined to estimate a population mean coefficient vector. Implementing this possibility is somewhat problematic because it requires an *a priori* basis for assuming some unknowns which are causing difficulties in the first place. Consequently, ultimate success of this operation requires selecting a stratification variable highly correlated with tree effects. Thus without much prior knowledge of how random effects could be correlated with potential stratification variables, some preliminary experimentation was necessary.

While several possible variables were considered and used in preliminary analyses as a stratification basis, tree crown class (dominant, codominant, intermediate, and suppressed) was found to be an adequate and simple choice. This stratification basis was used for all three structural models.

Tree crown class classification is somewhat subjective and it was noted that residual trees are often reclassified into higher crown classes after logging. Consequently, on logged-over plots, classifications prior to harvest were used for stratification.

5.3.2 Estimation Methods and Problems

Given the data sources available for model development, one means of accomplishing the previously described development objective would be the following procedure;

1) Assume the variation in model coefficients between trees within a crown class is negligible. Each stratified data set can then effectively be considered a time series with direct least squares producing consistent and efficient coefficient estimates to be used in a recursive type A growth model for trees in the stratum.

2) Fit the appropriate growth model to each crown class. This will produce

$$SS_{lf} = \sum_{j=1}^{n_l} [y_{lj} - f(x_{lj} , \hat{\theta}_l)]^2 \qquad (5\text{-}1)$$

where

θ_l = vector of estimated coefficients minimizing SS_{lf}

lj = denotes the j^{th} tree in crown class l, l = 1,4.

f = represents any of the three increment functions.

x_{lj} =vector of explanatory variables that determine the level of "f".

y_{lj} = growth of the j^{th} tree in crown class l.

SS_{lf} = sums of squared residuals for crown class l.

n_l = number of sample trees for crown class l.

3) If each of the four crown class parameter vectors are partitioned into two subvectors, say $\theta 1_l$ and $\theta 2_l$, we would like to determine if the individual crown class estimates of $\theta 1_l$, l=1,4, are all independent estimates of the same subvector $\theta 1$. This can be stated as an hypothesis H_o.

$$H_o: \quad \theta 1_1 = \theta 1_2 = \theta 1_3 = \theta 1_4 = \theta 1$$

If we let $\hat{V}(\hat{\theta}1_l)$ be the estimated variance-covariance matrix of the subvector of estimates $\hat{\theta}1_l$, l = 1,4, a pooled estimate of the mean subvector $\theta 1$ can be obtained from

$\frac{\sum (\hat{V}\hat{\theta}1_l^{-1} \hat{\theta}1_l)}{\sum V\hat{\theta}1_l^{-1}}$

$$\tilde{\theta}1 = [\sum_{l=1}^{4} \hat{V}(\hat{\theta}1_l)^{-1}]^{-1} \sum_{l=1}^{4} \hat{V}(\hat{\theta}1_l)^{-1} \hat{\theta}1_l \qquad (5\text{-}2)$$

Swamy (1970) has shown that under H_o, (5-2) is an efficient and unbiased estimate of $\theta 1$ if the explanatory variables (x_{lj}) are non-stochastic. With stochastic regressors, the estimate is consistent and asymptotically unbiased (Maddala, 1977).

4) A model with $\Theta 1$ constrained can subsequently be fitted to each crown class to obtain

$$SS_{lc} = \sum_{j=1}^{n_l} [y_{lj} - f(x_{lj}, \tilde{\Theta}1, \hat{\Theta}2_l)]^2 \qquad (5\text{-}3)$$

where

$\tilde{\Theta}1$ = is obtained from (5-2) and fixed in the model prior to estimating $\Theta 2_l$.

$\hat{\Theta}2_l$ = estimated subvector minimizing SS_{lc}

SS_{lc} = constrained sums of squared residuals for crown class l.

5) 5) A likelihood ratio can be computed as

$$T = \frac{\sum_{l=1}^{4} SS_{lc}}{\sum_{l=1}^{4} SS_{lf}} = \frac{SS_c}{SS_f} \qquad (5\text{-}4)$$

If H_o is true, we would expect T to be close to unity and we could conclude that the individual crown class estimates $\{\Theta 1_l\}$ are all independent estimates of the same subvector $\Theta 1$. Gallant (1975) describes the sampling distribution of T and suggests approximate tests based on critical points of a central F-distribution. Unfortunately, SS_f has four components that are not totally separable. The component sums of squares are associated with the following sources:

SS_w = within crown class time-invariant trees effects

SS_u = time-variant effects among trees

SS_m = measurement error effects.

Similarly, SS_c has the same components with the addition of

SS_g = between crown class tree effects

What is desirable to reflect in the statistic T under H_o is

$$T = \frac{SS_g + SS_u}{SS_u}$$

but what is actually being incorporated is an estimate of

$$T^* = \frac{SS_w + SS_g + SS_u + SS_m}{SS_w + SS_u + SS_m}$$

Total variation due to time-invariant tree effects is symbolically represented as $(SS_w + SS_g)$. Hence, if the basis for stratification is ill-chosen, SS_w will be large relative to SS_g and this will tend to deflate the computed test statistic T^* relative to T[1/]. At this stage in model development, there is no direct method to test if the basis for stratification is adequate.

A second major problem is that measurement error is unavoidably an added source of contamination which also contributes to underestimating the test statistic. While difficult to assess directly, indirect methods (Krumland and Wensel, 1981) suggest that variation due to measurement error in diameter increment models is of the same magnitude or larger than time variant tree growth variation. The relative impact of measurement error on the height growth data is suspected to be even larger. All of the measurements of height growth have been derived from successive differences in total height measurements taken on two occasions. In general, measurement techniques involved measuring ground distances and, subsequently, using a hand held clinometer to measure

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[1/] If the hypothesis outlined in Chapter 3 concerning correlations of tree effects with size variables is true, a compounding problem is that the individual stratum coefficient estimates will also be biased.

total height. In several instances, measurements were rounded to the nearest five feet which is about the limit of accuracy of clinometers on trees in excess of one hundred feet tall. Coupled with the fact that it is often difficult to even see the tops of trees in coastal stands and, once trees achieve heights of over 150 feet, five-year height growth is roughly of the same magnitude as the rounding fraction, it was not too surprising to find that about 20% of the height growth measurements were either negative or over 2.5 times the rates indicated by site index curves.

Additional secondary problems stem from the representative nature of the sample. While trees could be randomly drawn within crown classes to provide an observational data base for estimation, the sampling proportions of adequately measured trees in each crown class are not necessarily a reflection of the population proportions.[2/] This problem, however, can be remedied by a suitable system of weights.

Lastly, even though the previous problems render any formal tests practically useless, the hypothesis H_o is a rather restrictive one. We might be willing to assume certain parameters are invariant between trees for the sake of simplicity in model operation even though some formal test might indicate they are statistically different. The question of how much variability is acceptable does not seem answerable on any _a priori_ grounds.

As a result of these problems, conventional tests of inference based directly on summary statistics derived from the previously

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[2/] This was unavoidable as the numbers of adequately measured suppressed and intermediate trees were much smaller than codominant and dominant crown classes.

described estimation procedures would be misleading. Consequently, some subjective judgment is unavoidable in implementing the model system. However, an indirect evaluation of these judgments can be made in the validation stage of model development.

5.4 Model Development Strategies and Sample Selection

5.4.1 Specification of Functional Forms

Development of functional model forms began with several explanatory variables and a general idea of the direction and magnitude of their effects on tree growth. A variant of the open grown tree concept was considered to be a reasonable approach in constructing models. As this approach assumes the various factors influencing tree growth interact in a multiplicative manner, the resulting forms tend to be non-linear in some of the parameters. The availability of derivative-free non-linear estimation packages,[3/] however, provides the somewhat dubious capability of constructing an almost unlimited number of explicit model forms. To make the task manageable, a preliminary computer sorting procedure was developed to aid in model construction. As input, this program accepts a series of ranges for each potential independent variable. For each possible combination of range classes, the program performs an intersection on the data and computes the average value of the appropri-

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3/ All non-linear parameter estimation in this report was accomplished with the IMSL subroutine ZXSSQ (IMSL reference manual, LIB-0007, edition 7, 1979, Houston.) This subroutine was imbedded in a larger overlay routine prepared by the author which was used to summarize the estimation results and develop statistics comparable to the output of standard linear regression packages. A similar overlay routine utilizing the IMSL subroutine RLSTEP and related software was developed for linear least squares parameter estimation.

ate (dependent) growth variable. The net effect of this procedure is to essentially hold all other variables constant and provide some indication of the effects of one variable on growth. The program was also used to examine interactions among independent variables. This screening process provided an initial basis for developing explicit model forms.

5.4.2 Sample Selection

In general, the data available for modelling can be characterized as having an excessive number of inappropriate growth observations that are contaminated with measurement errors. In addition, subsamples of the data fitted to preliminary model forms indicated the residual variances are heteroscedastic and unequal between trees in different crown classes. Further, graphically fitting models with data composed of cell means computed by the data sorting program produced models that seemed more logical and performed better in simulations than models based on a least squares fit to a subsample. Consequently, the following procedure was followed to produce sample sets for subsequent statistical analysis:

1) The relevant independent variables were selected based on preliminary screening with the data sorting program.
2) The entire data base was recompiled, sorted, and all the "cells" with 3 or less observations were deleted. For cells with three to nine observations, the median growth was used as the growth response for the cell. For cells with 10 or more observations, residuals from the mean growth were ranked and approximately ten percent of both the largest positive and negative deviations were deleted. The "trimmed mean" (Bickel and Doksum, 1977) was then used as the growth response for the cell. Use of the median and

trimmed means are considered "robust" estimators and tend to be much more efficient than the mean in the presence of outliers (Huber, 1981)

3) This screened and aggregated data base was then used for model fitting. Values for the independent variables were taken to be the range midpoints. A description of the variables and ranges used for each of the following models is included in Appendix II.

There are several benefits that can be realized by following the previous procedure in model development versus using a subsample directly in estimation:

1) It reduces the data base from several thousand observations to 100-400 "data" points, depending on the species and growth model. Hence, the entire data base can be used economically in subsequent analysis.
2) The heteroscedastic nature of the data is virtually eliminated.
3) The adverse effects of outliers is substantially reduced.
4) Residuals from the fitted model are conveniently reanalyzed by the data sorting program to examine reasonableness of model fits.

While "standard errors" and other statistics computed from this type of data are unsuitable for formal tests of inference that might be used in choosing which coefficients in the model system might be random or determining the best functional form, it is emphasized that the unaggregated and untrimmed data has similar or even more severe defects.

5.5 Total Height Increment Models

Most historical research in height growth has centered around the development of site index curves. It is generally recognized that height growth, particularly of dominant and codominant trees is much less sensitive to changes in competition and crown size than is diameter growth. Hence, the primary determinant in estimating future height growth is based on cumulative past height growth of a group of "site trees" in a given location and is called "site index". The site index models used in this study were developed for redwood by Krumland and Wensel (1977a) and conversions of site index equations of other species to this model form are described in by Krumland and Wensel (1979).

5.5.1 Structural Component Development

The height growth model uses the five year change in total height in feet (yh) as the dependent variable. The general form of the structural component is

$$f_h = \text{(potential)(competition factor)}$$

5.5.1.1 Height Growth Potential

When site index equations are based on time series data, they provide reasonable estimates of the growth trajectories of individual dominant and codominant trees. Hence, in view of the problems associated with the currently available data, site index equations were used as an integral part of the growth models to ensure some reasonableness in model performance. The following conventions and procedures are used:

(a) Site index equations give total height in feet (HT) of dominants as a function (g_h) of site index (S) and breast high age (BHA). Implicitly,

$$HT = g_h(S,\ BHA)$$

(b) Manipulate the basic site index equation to express age as a function (g_a) of height and plot site index.

$$BHA = g_a(S,\ HT)$$

(c) For each tree, we tentatively assume it is a dominant and get an "estimated" breast high age (EBHA) by using its current height and site index.

(d) If the tree is a dominant, its five year height growth (DHG5) could be estimated as

$$DHG5 = g_h[S,\ (EBHA + 5)] - HT$$

Using DHG5 as a prediction of height growth will virtually replicate a site index curve if site index and initial height are known. As a whole, trees grow somewhat less than dominants and at some point, reductions in live crown ratio begin to have a negative impact on height growth. The "potential" portion of the height growth model was subsequently specified as

$$\text{height growth potential} = \alpha_1 DHG5/\{1. + \exp(4 + \alpha_2 CR)\}$$

where

CR = live crown ratio

α_i = coefficients to be estimated by non-linear regression

5.5.1.2. Height Growth Competition Factor

Experimentation has shown that a reasonable form for the height growth competition factor is the following logistic function of canopy closure at 66% (CC_{66}) of total tree height.

$$\text{height growth competition factor} = 1/\{1 + \exp(-3 + \alpha_3 CC_{66})\}$$

In subsequent estimation, initially treating the fixed coefficients (4 and -3) in the model as parameters indicated a very high degree of correlation between the estimates for them and the parameter α_1. Fixing their values at an *a priori* basis resulted in no loss of explanatory power and made extrapolations to the extreme limits of applicability more stable.

5.5.2 Sample Description

The height growth sample consisted of all trees with total heights, height growth, and crown ratios measured in the data base. These data were aggregated by the procedures described in section 5.4.2. Number of trees by species and crown class are shown in table 2. Data ranges are described in Appendix II.

Table 2. Numbers of sample trees and data cells with three or more trees by species and crown class used in height growth model development.

Species	Dominants	Codominants	Intermediates	Suppressed
	(No. of Trees)/(No. of Cells)			
Redwood	236/12	392/23	255/19	131/16
Douglas fir	205/13	191/15	106/14	35/7
Tanoak	0	0	0	0
Alder	0	0	0	0

5.5.3 Estimation Summary

Initially fitting the model separately by species to each crown class data set indicated the separate coefficient estimates for α_1 varied considerably. The estimates α_2 and α_3, however, were numerically similar for both redwood and Douglas fir except in the suppressed crown class for redwood and both the intermediate and suppressed crown classes for Douglas fir.

In these three categories, variation in live crown ratio was minor which presumably resulted in a high degree of correlation between the parameter estimates of α_1 and α_2. Fixing the value of α_2 at pooled values from the separate crown class regressions produced estimates of α_3 that were numerically similar for dominant, codominant, and intermediate trees for both species although the parameter estimates for α_3 for both suppressed data sets indicated no significant response to competition and estimates of α_1 seemed to be abnormally low. This was attributed to the small degree of variation in CC_{66} for these crown class groups as well as the small number of cell aggregates. Also, individual regressions accounted for less than 8% of variation about the mean for the suppressed tree data sets. In view of these somewhat indecisive results and the fact that suppressed trees represent a minor component of most stands, the parameters α_2 and α_3 were assumed to be invariant between trees, all four crown class data sets for a given species were combined, and the following functional form was used in estimation:

$$f_h = \frac{[\sum_{k=1}^{4} \alpha_{1k} x_k] \; DHG5_{li}}{[1 + \exp(4 + \alpha_2 CR)][1 + \exp(-3 + \alpha_3 CC_{66})]} \tag{5-5}$$

where

li = the i^{th} tree group in crown class l, $l=1,2,3,4$

x_k = 1 if $k=l$,

= 0 otherwise.

α_{1k} = crown class specific coefficients

Coefficient estimates were obtained by weighted non-linear least squares where the weight for each "data" point was the number of trees in the group. For comparative purposes, the four crown class data sets were combined and coefficients for a three parameter model (the α_{1k} were replaced by a single coefficient in (5-5)) were estimated. Table 3 shows the proportional difference in the sums of squared residuals (SSR) from this model and (5-5) when compared to the pooled SSR of fitting the model separately to each data set. There is only a marginal increase with (5-5) but a substantial increase when crown class specific parameters are replaced with a single parameter. These results are considered to indicate that treating α_1 as random coefficient and the remaining coefficients as constants is a reasonable characterization of height growth.

Table 3. Proportional increases in residual sums of squares from model (5-5) and the restricted three parameter model to the pooled sums of squares from separate fits to each crown class.

Species	Form (5-5)	3 Parameter Model
Redwood	.04	.33
Douglas fir	.05	.27

Implementation of the model system requires a single coefficient α_1 in place of the $\{\alpha_{1k}\}$ as estimated with (5-5). To provide an estimate that represents the "population average", the proportions of trees in each crown class on all available growth plots were determined for each species. These proportions were averaged over all plots to provide estimates of population proportions. Then, α_1 was estimated as

$$\hat{\alpha}_1 = \sum_{l=1}^{4} \hat{\alpha}_{1l}\hat{p}_l$$

where $\hat{p}_l$ is the estimated proportion of trees in crown class l in the data. To illustrate the relative magnitudes of the individual crown class coefficients, the $\{\hat{\alpha}_{1l}\}$ were subsequently expressed as proportional deviations from $\hat{\alpha}_1$. All of the coefficient estimates are shown in table 4.

Adequate data sets were unavailable for the hardwood species. As a temporary measure that can be remedied when better data are available, redwood coefficients were used for tanoak and Douglas fir were used for alder in model operation.

Table 4. Estimated coefficients for the height growth models. $(\alpha'_{11} = (\alpha_{11} - \alpha_1)/\alpha_1)$

Species	α_1	α'_{11}	α'_{12}	α'_{13}	α'_{14}	α_2	α_3
Redwood	.935	.235	.185	-.031	-.171	-25.5	1.35
Douglas fir	1.050	.171	.157	-.121	-.240	-27.3	1.28

Trial simulations with the height growth coefficients and the individual crown class estimates of α_1 used as preliminary equation modifiers (in conjunction with the crown recession models described in the next section) indicated the average heights of dominant and codominant trees resulting after several decades of simulations is centered around values obtained by site index curves. Hence the general level of the estimates are consistent with observed growth. A more detailed evaluation is described in chapter 6.

5.6 Crown Recession Models

Crown length and crown ratio relationships play a major role in the DBH and total height increment equations. Consequently, crown recession models are a fundamental component to the model system, particularly when yield predictions are being made for several decades. There have been no apparent direct attempts to develop crown change models with the dependent variable being change in height to the crown base. Other modelers have used indirect methods such as (1) assumed branch mortality (Mitchell,1975); (2) estimating crown ratios from other stand variables (Holdaway et.al.,1979, Daniels et.al.,1979); and (3) developing crown length estimators and partially differentiating the equation so presumed change in crown length is a function of changes in other variables such as tree height (Stage, 1973).

These attempts were probably motivated out of necessity as adequate data bases on crown recession are universally scarce. Attempts to use indirect methods in modelling this variable were abandoned because of ambiguities and inconsistencies in application. For example, if the crown ratio on individual trees is estimated as a function of stand density, a harvest wouldn't actually result in an immediate effect on crown ratio of the residual trees but the predictions would indicate it had. A direct attempt was made to model crown recession based on data derived solely from Jackson State Forest CFI plots. A description is provided in Appendix I. Data were aggregated by the procedures described in section 5.4.2 and the numbers of trees by species and crown class are shown in Table 5. Ranges used to aggregate the independent variables are shown in Appendix II.

Table 5. Numbers of sample trees and data cells with two or more trees by species and crown class used in crown recession model development.

Species	Dominants	Codominants	Intermediates	Suppressed
	(No. of Trees)/(No. of Cells)			
Redwood	112/15	196/19	104/16	78/12
Douglas fir	85/12	66/13	43/8	22/4

Modelling crown base recession presents a challenge because of several apparently reasonable but contradictory observations that can be made:

1. In general, crown bases are higher in dense stands than in moderately stocked ones. Presumably, crown recession rates were faster in the dense stands. There is less light penetration to the

lowermost branches and this may accelerate needle mortality.

2. Within stands, there is presummably a gradient of light availability that increases with height. Height to crown base in intermediate and suppressed trees is usually less than in the dominant-codominant stand fraction. So it would seem that even though the suppressed trees have less light which would tend to be positively correlated with the crown recession, actual change in height to the crown base is less for these trees than the somewhat lesser light stressed dominants. shorter than dominants due

5.6.1 Structural Component Development

At the risk of oversimplification, the following scenario is offered to provide a basis for model development. Trees with long crowns tend to be more sensitive to light competition than trees with shorter crowns. The lowermost branches on long crowned trees contribute proportionately more to branch maintenance than to bole growth and do not seem to be vital to the trees existence. In shaded conditions, net photosynthesis in these branches may be negative and consequently, they are somewhat more dispensable than lowermost branches on short crown trees. Trees that are growing rapidly in height also tend to have faster rates of crown recession. Inspection of undisturbed evenaged stands indicates a somewhat uniform crown base line through the dominant-codominant stand portion. However, crown bases on the noticeably taller trees are somewhat higher even though the overall crown length may be greater. Presumably, faster growing regions of the tree (particularly those above the main canopy) use much more water at the expense of supplying water to the lowermost branches. This may accentuate crown base recession.

While not being a total biological representation, the following functional form was selected after graphical screening to predict five-year change in height to the crown base (yc) and was used as an initial basis for analysis.

$$f_c = \frac{c_1[1 - \exp(c_2 Cl)] + c_3 \hat{y}h^{c_4}}{1 + \exp[c_5 + c_6 CC_{htcb}]} \qquad (5\text{-}6)$$

where

CL = current crown length

$\hat{y}h$ = estimated population mean five year height growth obtained from (5-5)

CC_{htcb} = estimated canopy closure percent at the crown base.

c_i = coefficients estimated by non-linear regression.

5.6.2 Estimation Summary

This model was initially fit separately to the individual crown class data sets with poor results[4/]. The parameter estimates resulted in certain explanatory variables having numerically insignificant impacts on predictions or having an effect opposite to what was expected. This was particularly noticeable in the competition related coefficients c_5 and c_6. Several attempts to develop a simpler model form resulted in simulated crown development that was often illogical and uncharacteristic of the the distribution of crown sizes observed in coastal stands.

4/ Some of the individual crown class data sets lacked enough grouped data points to even attempt an estimation.

Combining the individual crown class data sets resulted in some improvements but the predictions remained insensitive to competition levels.

After extensive experimentation, it was found that using predicted height growth for each crown class, (model (5-5) with the coefficient estimate of α_1 replaced by the estimates of α_{11}), resulted in parameter estimates that seemed superficially reasonable. Also, the predicted distribution of crown sizes in even-aged stands resulting after simulations several decades in length were in conformance with observed patterns in older stands. The model (5-6) was subsequently fit with this convention to different species. Coefficient estimates are shown in table 6 for redwood and Douglas fir. Since no data were available for other species, the same coefficient substitutions made with the height growth models were used here also.

Table 6. Coefficient estimates for the crown recession models by species.

Species	Coefficient Estimates						Mean Growth (feet)
	c_1	c_2	c_3	c_4	c_5	c_6	
Redwood	5.50	-.031	.69	.82	2.73	-3.03	2.81
Douglas fir	4.12	-.130	.32	.97	4.10	-5.20	5.03

In terms of the "modifier" components of the model outlined in chapter 2, all of the possible time invariant random effects in crown recession were assumed to be propagated by assigned random effects in height growth. This assumption, the validity of which is not directly testable, greatly simplifies the development of stochastic schemes for full model operation.

5.7 Diameter Increment Models

The total height increment models used an extraneously derived, and presumably reasonable, estimate of an individual tree growth trajectory as a primary component with only the level being treated as random between trees. Unfortunately, no similar devices were available for diameter increment model development. There are several other factors which dictate that special attention be given to the development of these model types:

1) It is generally recognized (cf., Spurr, 1952; Staebler, 1963) that changes in tree bole increment are much more sensitive to crown reductions and competition than is height growth.
2) The predictions of tree volume used in this study are more sensitive to changes in tree diameter than total height.
3) Data that might be available for post calibration of these models is practically limited to past measurements of diameter increment.

5.7.1 Structural Component Development

The same general growth relationship that was used for height increment models was used here in the specification of a functional form.

$$f_d = (\text{potential})(\text{competition factor})$$

5.7.1.1 Diameter Growth Potential

After extensive experimentation and simulations based on a preliminary implementation of the entire model system, the following functional form was adopted as the potential change in tree DBH squared.

$$\text{potential} = \delta_1 S^{\delta_2} HT^{\delta_4} \ln[\delta_3 / HT^{\delta_4}][1 - \exp(\delta_5 CR)]^{\delta_6} \qquad (5\text{-}7a)$$

where

S = plot site index in feet for the species under analysis

HT = total height in feet

CR = crown ratio

ln(x) = natural logarithm of "x".

δ_j = regression coefficients

Experimentation with several alternative forms indicated comparable predictions throughout stand age ranges of 30-80 years where the bulk of the sample data was obtained. However, they produced predictions which were considered to be excessively high in young stands (10-20 years old) or very old young growth stands (70-90 years old) or else resulted in slight oscillations in stand basal area growth in ages above 80 years. While these oscillations are probably within the range of reliability of the model, they tend to be disconcerting to potential users who are accustomed to the prevailing theory of a monotonically decreasing stand increment with age.

5.7.1.2 Diameter Growth Competition Factor

Use of the graphical methods described earlier and experimentation with several variables led to the adoption of the form

$$\text{competition factor} = 1 + \left(\frac{2}{\pi}\right)\text{Atan}[\delta_7 \pi(-CC_{66}) CL^{\delta_8}] \qquad (5\text{-}7b)$$

where

CC_{66} = Estimated canopy closure at 66% of total tree height

CL = live crown length in feet (total height times crown ratio)

Atan(.) = arctangent of "."

π = 3.1415...

δ_j = coefficients to be estimated

The arctangent function was chosen to represent the effects of competition because preliminary graphical analysis indicated it could fit plotted trends very well. Several attempts were made during analysis to develop a density index that was a weighted function of canopy closure at different percentages of tree height. Singly, CC_{66} produced the greatest marginal decrease in residual sums of squares when compared to other possibilities such as canopy closure at 40 percent of tree height or at the crown base. However, after the inclusion of CC_{66}, all other additions were either insignificant or had the wrong signs. This is probably due to the high degree of correlation between canopy closure percent at different heights on the same tree in a given stand or the fact that differences in canopy closure percentages evaluated at different percentages of tree height also tend to be correlated with tree height. Consequently, the model assumes that trees that are less than two thirds the height of the subject tree offer no immediate competition. While this assumption may be tenable, a more detailed analysis will require better data sources than are available to the current study.

Another noteworthy aspect of the competition component is the addition of the term involving crown length. Graphical analysis indicated that, for all other variables being held constant, the range in relative growth response to different competition levels is less for long crowned trees than for short crowned ones. While a biological argument might be constructed to support this observation, earlier versions of the model system (Krumland and Wensel, 1981), ignored this aspect and used a competition component that was independent of crown length. Several users of the earlier system remarked that predictions of pre-commercial thinning response of leave trees in young stands (short crownd trees) seemed low and the converse being apparent in older stands. The addition of crown length to the competition component of the model seems to have remedied the problem.

In subsequent sections, the product of (5-7a) times (5-7b) is referred to as (5-7).

5.7.2 Data Sets

Data sets were developed for these models similar to the ones constructed for height growth. One growth measurement was selected per tree that had corresponding height, crown class, and crown ratio measurements. These data were smoothed and aggregated by the methods previously described. The ranges used for aggregating are described in Appendix II. Numbers of sample trees by species and crown class are shown in table 7.

Table 7. Numbers of sample trees and data cells with three or more trees by species and crown class used in diameter increment model development.

Species	Dominants	Codominants	Intermediates	Suppressed
	(No. of Trees)/(No. of Cells)			
Redwood	566/72	4254/97	2004/107	1344/78
Douglas fir	775/49	1125/53	317/34	63/12
Tanoak	49/10	155/18	371/21	51/11

5.7.3 Estimation Summary

If we consider (5-7), the product of (5-7a) and (5-7b), to be a suitable form for the implicit diameter growth model

$$f_d(xd, \Theta d)$$

as described in Section 2.4, concern in this phase of estimation deals with (a) obtaining an estimate of the mean coefficient vector $\Theta d = \{\delta_1, \delta_2, \ldots \delta_8\}$ that is reasonable when (5-7) is used in a Type A framework, and (b) finding a reasonable partition (if any) of Θd into $[\Theta d1 \mid \Theta d2]$ such that the sample units can be considered homogeneous so far as $\Theta d1$ is concerned.

In developing model estimates, much of the exploratory analysis was performed with the Douglas fir data sets. This species group had the most ill-conditioned[5/] data and was considered to cause the most problems in estimation.

The model (5-7) was initially fit separately to the individual crown class data sets. As with the height growth models, the individual parameter estimates of δ_1 varied proportionately the most between crown classes, consistently decreasing from dominant to suppressed trees. There was an apparent high degree of correlation between the remaining parameter estimates due to the lack of within-crown class variation of some of the independent variables. The parameters δ_2, δ_7, and δ_8 seemed fairly stable across crown classes so they were pooled and the model refit in constrained form to each crown class data set. Increases in the residual sums of squares of the individual fits to separate crown classes were at most ten percent with these constraints. Further experimentation with different subsets of pooled parameter estimates indicated comparable results largely because, within a crown class, the

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5/ The variation in the mean height and crown ratio (the predominant explanatory variables in the potential portion of the model) between crown classes is greatest for Douglas fir.

model seemed to be overparameterized. (i.e., a simpler model form could fit the data for a given crown class equally well, but simulated growth patterns seemed to be ill-conditioned at the extremes of the data).

In view of these somewhat indecisive results, a limited combinatorial analysis was undertaken that employed the following constructs in place of the j^{th} parameter (j=1,2,,,8) in the model.

$$Q_j = \sum_{l=1}^{4} q_{jl} z_{jl} \qquad j = 1,8$$

where

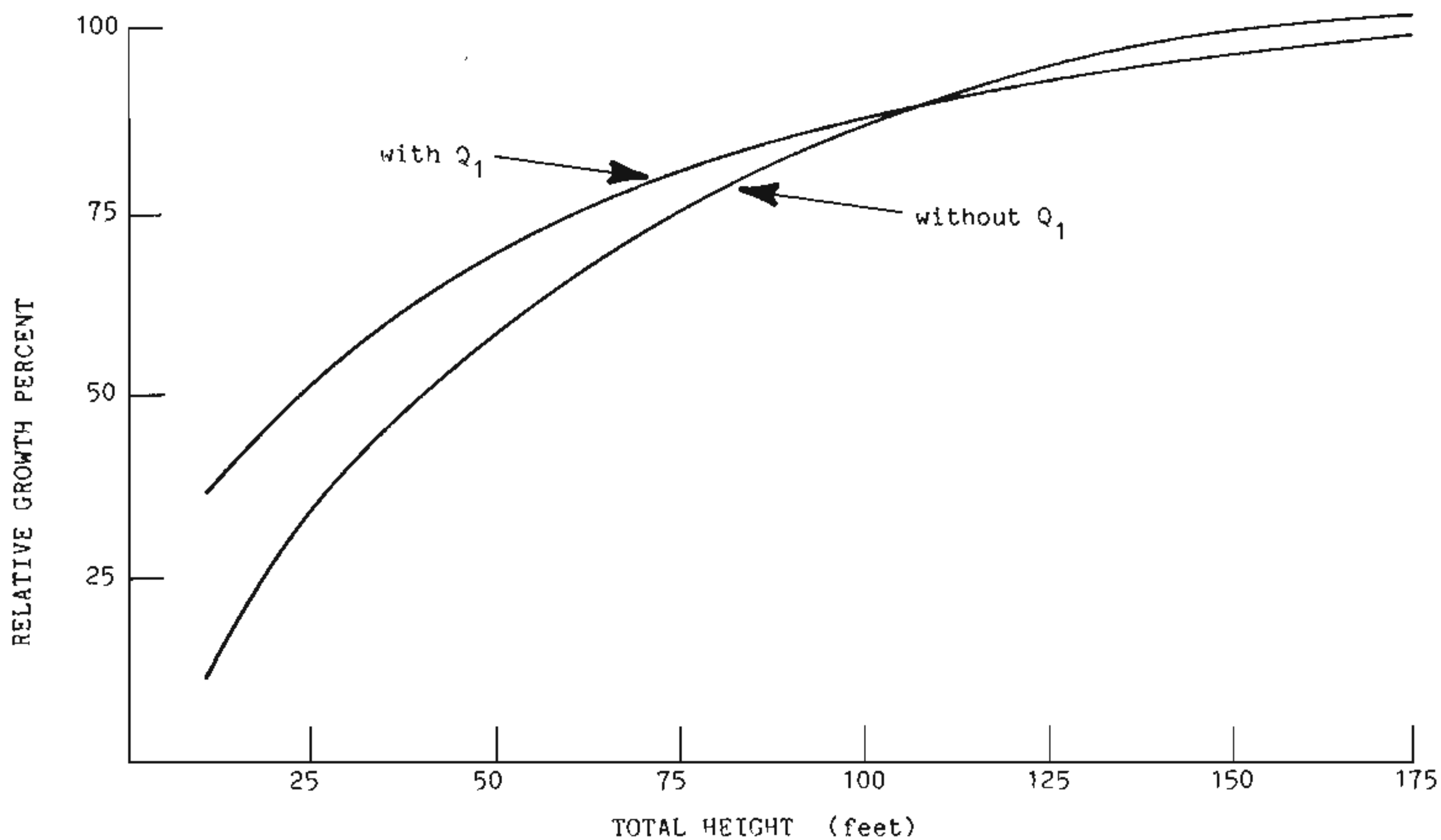
z_{jl} = "1" if the sample tree is from crown class "l", 0 otherwise.

q_{jl} = additional coefficients to be estimated representing the effect of a tree in the l^{th} crown class.

In separate estimations, the Q_j were substituted for the δ_j, either singly or in various combinations. The individual crown class data sets were combined and fit to (5-7) with these additions. The coefficients δ_2, δ_7, and δ_8 were fixed at their previously described pooled estimates during this phase of the analysis.

As the most likely candidate, Q_1 was initially substitiuted in (5-7) and resulted in the residual sums of squares increasing in the order of 5% when compared to the pooled estimates from separate crown class regressions. Further additions of pairs or triplets of Q_i produced only marginal reductions in residual sums of squares when compared to the model estimated with Q_1. Hence, it was assumed that treating δ_1 as random and the remaining parameters as fixed would be reasonable and also facilitate subsequent adjustments and calibrations.

As an indication of the difference in growth estimates obtained from fitting (5-7) by different methods, figure 7 shows the effects of



Firgure 7. Relative effects of tree height on diameter growth for Douglas fir for two different methods of estimation.

tree height on growth from models fit with and without Q_1 for Douglas fir. The differences between these models is largely due to the difference in mean height of different crown class data sets and is in accordance with the general adverse effects on model development that was outlined in chapter 3. Similarly, figure 8 shows the relative difference in the effects of crown ratio between the two models. Analyzing the residuals from both of these models with the aid of the data sorting program indicated no major trends in form fitted with Q_1 with any explanatory variable but the model fitted without Q_1 apparently overestimated the effects of crown ratio and underestimated the effects of density.

It was concluded that the diameter growth model could be characterized as having one random coefficient (δ_1) whose value varied from tree-to-tree and the remaining model coefficients would be constants for all trees. Subsequently, for a given species, all crown class data sets were combined, Q_1 was substituted for δ_1, and all the coefficients were reestimated by weighted non-linear least squares. Each data point was weighted by the number of observations making up an individual cell aggregate. The mean population coefficient, δ_1, was estimated as a function of the set of estimates $\{\hat{q}_{11}\}$ using the same crown class proportions and procedure that was used with the height growth model. The estimates of q_{11} were subsequently expressed as proportional deviations from δ_1. Table 8 shows the coefficient estimates for redwood, Douglas fir, and tanoak. Tanoak results indicate that dominants grow less in basal area then either codominants of intermediate trees. This is considered to be a species specific problem rather than a general defect of the methods employed in estimation. Tanoak tends to be classified as dominant in sparse or pure stand conditions. In these situations, much

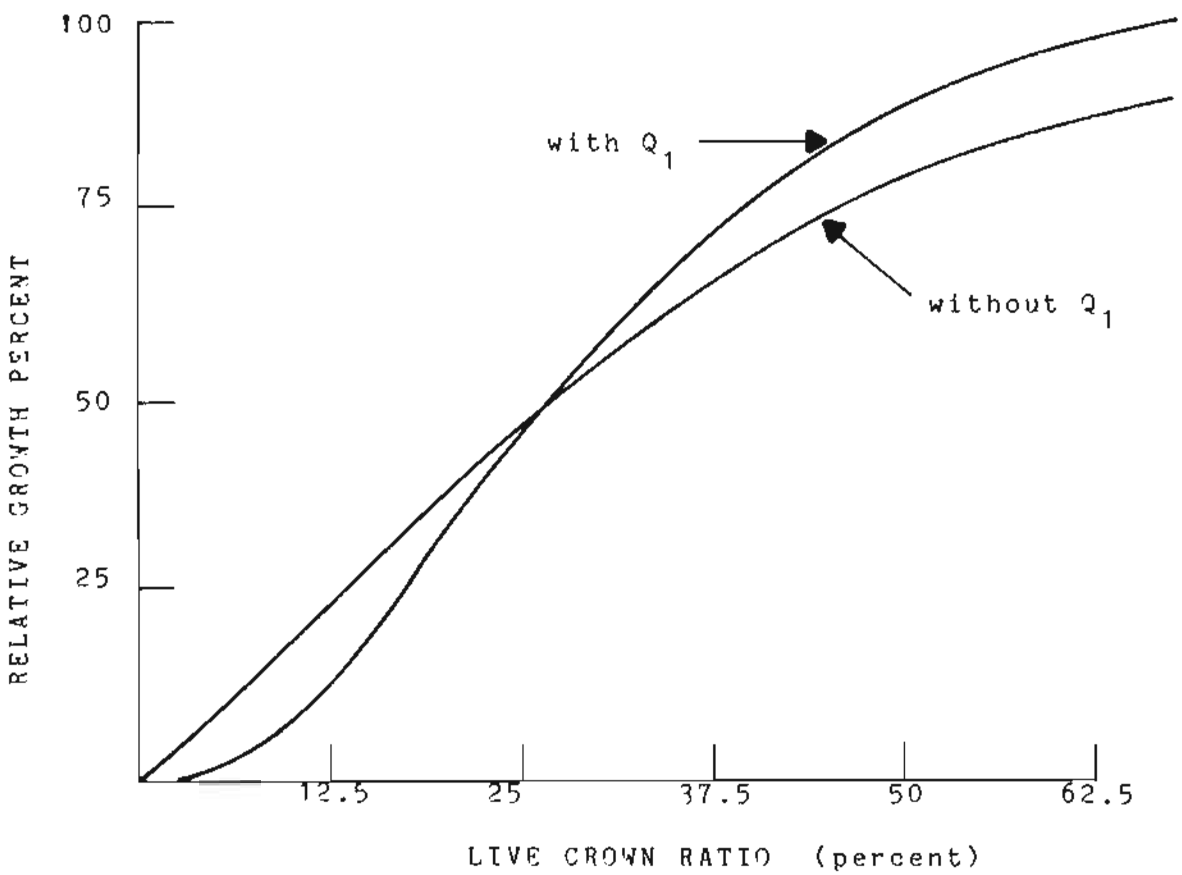


Figure 8. Relative effects of crown ratio on diameter growth for two different methods of estimation.

Table 8. Coefficient estimates of the diameter increment models by species ($q'_{11} = (q_{11} - \delta_1)/q_{11}$)

	Redwood	Douglas fir	Tanoak
δ_1	.0366	.0115	.160
δ_2	.804	1.04	.210
δ_3	260.2	140.4	4510.0
δ_4	.68	.73	1.70
δ_5	-1.61	-4.21	-4.46
δ_6	.76	1.74	3.24
δ_7	.82	.65	.51
δ_8	-.40	-.28	0.0
q'_{11}	.42	.36	-.37
q'_{12}	.11	.04	.44
q'_{13}	-.18	-.29	.43
q'_{14}	-.48	-.65	-.82

of the tree growth is concentrated in branch rather than basal area development and the trees resemble large bushes. In mixture with conifers, tanoak tends to be shorter and classified in subordinant crown positions. Branch development is inhibited and growth tends to be concentrated in height and the tree bole. Tree form subsequently tends to be more cylindrical. In any event, there doesn't seem to be much that can be done to account for this phenomena without much more detailed data.

Since there were insufficient data to determine separate coefficients for alder, the estimated coefficients for Douglas fir and tanoak were used as a base and manually adjusted on a judgemental basis so that simulated patterns of basal area growth looked similar to empirical yield table estimates for alder in Washington (Chambers, 1974). Alder was included in the model mainly for the sake of completeness but this is an obvious area of future research.

In summary, model (5-7) with the coefficient δ_1 treated as random was chosen to be the form for the diameter increment model. Estimation techniques employed produced simulated growth trajectories that are different from an ordinary least squares development based on pooled data from all crown classes. As these differences were anticipated and in conformance with the hypothesis stated in Chapter 3, the estimated model seems to be an improvement. A more detailed evaluation is described in Chapter 6.

5.8 Mortality Models

Mortality models were developed by Krumland et. al. (1978) that are of a form that is directly usable in the model system. As no additional data have become available since then, they were adopted as component models for this study.

5.9 Equation Modifier Development

With the exception of the coefficients α_1 in the height growth models and δ_1 in the diameter growth models, the remaining coefficients in the model system are considered non-random and the point estimates previously described are considered to be reasonable values to use in model operation. Subsequent development of the model system ignores any sampling error associated with these estimates and effectively treats them as known population parameters. In this section, relationships needed for equation modifier development are derived. The parameters needed to initialize the model system are subsequently estimated.

5.9.1 Modifier Forms and Purpose

With only two coefficients per species in the model system considered random, development of a modifier framework is simplified. For the diameter increment model, each tree has a random coefficient, $\delta_{1_{ij}}$, where ij denotes the j^{th} tree on the i^{th} plot. This random variable is related to the mean population coefficient (δ_1) by

$$\delta_{1_{ij}} = (1 + ad_i + bd_{ij})\delta_1 \tag{5-8}$$

where

ad_i = average percent deviation of all trees on plot "i" from δ_1 (plot effect).

bd_{ij} = percent deviation of tree "j" from ad_i (tree-within-plot effect).

The combined term

$$(1 + ad_i + bd_{ij})$$

constitutes the form of the equation modifier. In model operation, estimates, or "randomly" chosen values of ad_i and bd_{ij} are incorporated as multiplicative adjustments applied to the mean predictions. Hence, predicted tree growth is implicitly represented as

predicted tree growth = (modifier)(mean population prediction)

An analogous development is used for height growth coefficients.

$$\alpha_{1_{ij}} = (1 + ah_i + bh_{ij})\alpha_1 \tag{5-9}$$

While interest is ultimately in the combined plot plus tree-within-plot effects, a separation into these two components allows greater flexibility in calibrating the model (i.e., the entire level of the plot projection can be altered by changing only the plot random factor).

As indicated in section 2.4, incorporation of random variation in simulations, say through some form of assignment of different tree-within-plot effects to each record in the tree list, is desirable from the standpoint of producing realistic estimates of future tree size class distributions. Given a parametric form for the combined distribution of the sets, $\{ad_i\}$, $\{ah_i\}$, $\{bd_{ij}\}$, and $\{bh_{ij}\}$, and estimates of parameters of the distributions, a marginal but sufficient amount of information is available for this purpose. Assignments could subsequently be made through some form of a Monte Carlo method. However, the probable correlations of tree effects with size characteristics that caused estimation problems with the structural growth models can now be exploited to increase the effectiveness of a modifier assignment strategy. For example, we can conjecture that, say, for a given height and crown ratio, trees that are larger than average in DBH may ~~tend~~ tend to have values for $\delta_{1_{ij}}$ that are greater than δ_1. Mitchell (1975) provides a hypothetical example that indicates such relationships may be somewhat ambiguous, however the ambiguity is one of degree and may be reconciled by additional growth information[6/].

5.9.2 Models for Random Effects

The next challenge then is to develop some general auxiliary plot and tree-within-plot "calibration" functions for both the height and diameter growth models. These functions are accessed at the start of a simulation to predict plot and tree-within-plot effects for each tree record in the tree list. These functions are components of models with the following implicit forms;

$$ad_i = P_d(zd_i, \beta_d) + ad_i^*$$
$$ah_i = P_h(zh_i, \beta_h) + ah_i^*$$
$$bd_{ij} = T_d(wd_{ij}, \gamma_d) + bd_{ij}^*$$
$$bh_{ij} = T_h(wh_{ij}, \gamma_h) + bh_{ij}^*$$

where

P,T = implicit plot and tree within plot calibration functions.

z,w = vectors of explanatory variables that are functions of current tree size characteristics at the start of growth projection.

β,γ = vectors of calibration function coefficients to be estimated.

The sets $\{ad_i^*\},\{ah_i^*\},\{bd_{ij}^*\},\{bh_{ij}^*\}$ represent random effects that are unexplained by the calibration functions. Each of these four sets are assumed to be composed of independent random variables with zero expectations and variances denoted as $\sigma^2_{ad^*}$, $\sigma^2_{ah^*}$, $\sigma^2_{bd^*}$, $\sigma^2_{bh^*}$ respectively.

6/ Incorporating additional growth information is the subject of Chapter 7.

The coefficients in functions P and T should also rightfully be considered random. This possibility is examined somewhat in later sections. However, for initializing the model system when no growth information is available, coefficient estimates derived from the current sample data should be sufficient.

The items considered to be of main interest in the development of equation modifiers are the calibration functions and associated coefficient estimates, and the tree-within-plot variances. The following section describes estimates of these components. The actual scheme used to assign growth equation modifiers is described in Chapter 6.

5.9.3 Sample Based Estimates of Random Effects

The development of general calibration functions requires estimates of plot and tree-within-plot random effects to be used as dependent variables in model analysis. As these items are not directly observed, estimates were used instead. These estimates were based on residuals of the sample data used to develop the height and diameter growth models.

In general, growth in height or diameter of a tree (y) is modelled as

$$y_{ijk} = (1 + a_i + b_{ij})f\{x,\Theta\} + u_{ijk} + m_{ijk} \qquad (5\text{-}10)$$

where as before

ijk = j^{th} tree on the i^{th} plot in time period k.

a_i = plot effect

b_{ij} = tree-within-plot effect

u_{ijk} = time variant random error

m_{ijk} = measurement error

$f\{x,\Theta\}$ = some growth function

Plots of residuals from the fitted mean population growth models in unaggregated form indicated substantial heteroscedasticity with the residual error variance being approximately proportional to the square of predictions. Residuals were subsequently expressed as proportions of predictions and taken to be estimates of the sum of independent and identically distributed random variables each with zero expectations. In other words, we let v_{ijk} be the proportional difference of actual to predicted growth

$$v_{ijk} = \frac{y_{ijk}}{f(x,\ \theta)} - 1 = a_i + b_{ij} + \epsilon_{ijk} \tag{5-11}$$

where

$$\epsilon_{ijk} = (u_{ijk} + m_{ijk})/f(x,\ \theta)$$

represents the proportional difference due to measurement error and time-variant random error. This variable (ϵ_{ijk}) is assumed to have a zero expectation. The sample data used to estimate $\{v_{ijk}\}$ is composed of I plots with J_i trees per plot. As most trees were only measured once, the subscript k in (5-11) is superfluous. Plot effects were subsequently estimated as the average proportional difference of the growth of all trees on a plot from the mean model prediction.

$$\hat{a}_i = \sum_{j=1}^{J_i} v_{ijk}/J_i \tag{5-12}$$

This estimator would seem reasonable as both b_{ij} and ϵ_{ijk} are assumed to have zero expectations. As most trees were measured only once, b_{ij} cannot be estimated. All we can estimate is ~~is~~ the combined tree-within-plot random effect (b_{ij}) and the time variant errors (ϵ_{ijk}). Denote this term as t_{ij}.

$$\varepsilon_{ij} = v_{ijk} - \hat{a}_i \qquad (5\text{-}13)$$
$$= \delta_{ij} + \zeta_{ijk}$$

5.9.4 Plot Calibration Functions

The plot calibration functions are intended to provide an estimate of the plot effects (a_i) at the start of a simulation. In actual computation of the sample based set $\{\hat{a}_i\}$ used in the development of the functions, a weighting scheme had to be employed because not all trees on a plot had sufficient variables measured to be growth analysis trees in (5-12). The proportion of trees that could be used were skewed toward the dominant and codominant crown classes which tend to be the "better" trees. Consequently, for each plot, the average value of v_{ijk} was determined for each crown class based on available growth sample trees. These values were then multiplied by the plot crown class proportions and summed to produce an estimate of a_i.

Experimentation with several variables led to the adoption of a simple linear model for the form ($p(z,\beta)$) of the plot calibration function

$$P(z,\beta) = \beta_o + \beta_1\left(\frac{\overline{HT}_i}{S_i}\right) \qquad (5\text{-}14)$$

where

$\overline{HT}_i$ = average height of all trees of the species under analysis for plot i at the time of measurement.

S_i = plot site index for the species under analysis.

This model form was used for both the height and diameter plot calibration functions. Coefficient estimates were obtained by linear least squares and are shown in table 9. Table 10 shows the mean square difference about the mean $\hat{a}_i$ of the data used to estimate the coefficients in (5-14) and the mean square residual from the fitted regression model. The plot calibration functions do not account for much of the variation between plots. However, the signs on the coefficients are as expected and inspection of the predictions indicate that average trees in young stands (15-20 years of age) are growing about five percent less than the model norm. The converse is true in older stands (80-90 years of age).

The root mean squares about the sample means of $\hat{a}_i$ are a rough approximation of the coefficient of variation of 5-year plot increment. For the diameter increment models (which is proportional to plot basal area increment) these values are about 30%. These estimates are of the same magnitude as other modellers have obtained from modelling periodic plot basal area increment of pure redwood and Douglas fir stands (Chambers, 1980, Lindquist and Palley, 1967) and would indicate that the precision of the individual tree models in aggregate are at least comparable to results that might have been obtained by other methods.

Table 9. Number of plots and coefficient estimates for plot calibration functions by species

	Height Growth β_o	Height Growth β_1	No. of Plots	
Redwood	−.12	.16	241	(−.072, .12)
Douglas fir	−.23	.27	68	(−.15, .175)
Tanoak	–––	–––	–––	

	Diameter Growth β_o	Diameter Growth β_1	No. of Plots	
Redwood	−.17	.23	379	(−.10, .175)
Douglas fir	−.21	.28	172	(−.126, .21)
Tanoak	−.03	.04	42	

Table 10. Mean squares resulting from fitting plot calibration functions to $\hat{a}_i$.

Source	Height Growth Redwood	Height Growth Douglas fir
Totals	.112	.094
Residual	.101	.083
Proportional reduction due to regression	.096	.120

Source	Diameter Growth Redwood	Diameter Growth Douglas fir	Diameter Growth Tanoak
Totals	.114	.079	.215
Residual	.099	.075	.211
Proportional reduction due to regression	.12	.05	.02

5.9.5 Tree-within-plot Calibration Functions

Tree-within-plot calibration functions are used to estimate b_{ij} for tree records in a tree list at the start of a simulation. Estimates of the coefficients of these functions were based on the sample data used to construct the structural growth models. This created a problem because direct estimates of tree-within-plot effects (b_{ij}'s) were available for only a small proportion of the data set as most trees were measured for growth only once. Hence, using residuals from the fitted structural model, we can only estimate t_{ij}; the tree effect (b_{ij}) plus the combined measurement and random error term (ϵ_{ijk}).

$$\hat{t}_{ij} = v_{ijk} - \hat{a}_i$$
$$= \hat{b}_{ij} + \epsilon_{ijk}$$

However, it would seem reasonable to assume that the combined measurement and random error term (ϵ_{ijk}) is uncorrelated with any size characteristics such that the following model would provide a suitable basis for analysis in estimating the coefficients of the tree-within-plot calibration functions

$$t_{ij} = T(w_{ij}, \gamma) + b^{*}_{ij} + \epsilon_{ijk}$$

In analysis, estimates of t_{ij} can be computed by (5-13) and used as independent variables in estimating the coefficients (γ) of the function T. The error terms in this model are now composed of the tree-within-plot effects unexplained by the calibration function (b^{*}_{ij}) plus the combined measurement and random time-variant error which is unaffected by the calibration.

Diameter Increment

While crown classification would be one avenue to pursue for tree-within-plot calibration functions, it was noted earlier that crown classification is somewhat subjective, tends to change after partial harvests, and is somewhat ambiguous in multi-storied stands for the purposes desired here. Consequently, it was felt that methods based on measurable size characteristics would provide a more objective basis for tree-within-plot calibration function development.

After graphical analysis with the aid of the data sorting program and the response variable being ε_{ij}, the following form was adopted for the tree-within-plot diameter calibration function.

$$T_d(wd_{ij}, \gamma_d) = \gamma d_1 hd_{ij} + \gamma d_2 cr_{ij} \qquad (5\text{-}15)$$

where

hd_{ij} = ratio of (total height −4.5) to DBH centered to the plot mean of a given species.

cr_{ij} = live crown ratio centered to the plot mean of a given species.

γd_i = coefficients to be estimated.

All trees from all plots were combined and the coefficients in (5-15) were obtained by linear least squares. Sample sizes and resulting coefficient estimates are shown in table 11 for each individual species.

Table 11. Sample sizes and estimates of the coefficients for the within plot diameter growth calibration functions by species.

Species	γd_1	γd_2	Plots	No. Trees
Redwood	-.42	-.46	346	8640
Douglas fir	-.19	-.807	123	1722
Tanoak	-.08	.01	24	384

Height Growth

Largely because height growth data of sufficient detail will usually be unavailable for post calibration of the model system, it was felt that expressing bh_{ij} as a function of bd_{ij} and using predicted values of bd_{ij} as access variables, whether they are estimated with the coefficients developed for (5-15) or result from "local" tree-within-plot post-calibration functions would be satisfactory operating convention. This would also reduce the impacts of any non-zero covariances between the height and diameter tree-within-plot effects that are unexplained by the calibration functions and facilitate assuming that they are effectively zero.

Using methods to develop a tree-within-plot height growth calibration function based directly on estimates of $\hat{td}_{ij}$ and $\hat{th}_{ij}$ obtained with (5-13) was not considered as the former is an estimate of the diameter growth tree effect plus measurement and time variant random errors. Hence, such an analysis can be viewed as an errors in variables problem (Johnston, 1963) with direct least squares resulting in biased coefficient estimates. A two stage approach was initially attempted using predicted values of bd_{ij} obtained from (5-15) instead of computed values of $\hat{td}_{ij}$. However, the measurement error component of $\hat{th}_{ij}$ seemed to be substantial and resulted in fitted models that produced unstable and sometimes illogical predictions. Consequently, some form of grouping and smoothing of the sample data was considered to be desirable before any further estimation was attempted.

Of several possible indirect procedures that were tried, the following produced fitted models that performed satisfactorily in model operation.

a) Within each plot, trees were stratified by crown class into four groups.

b) For each group, median values (or "trimmed means" if there were more than ten trees per group) were computed for δd_{ij} and δh_{ij}.

c) The resulting set of four "data" pairs per plot were subsequently used as sample set of bd_{ij} and bh_{ij} for subsequent modelling.

Graphical plotting of this set of variables as well as inspection of the crown class indicator variables estimated for the structural height and diameter growth equations indicates the relationship between them is not linear. The following functional form was subsequently used for the tree-within-plot height growth calibration function.

$$T_h(wh_{ij}, \gamma_h) = \frac{\gamma h_1}{1 + \exp(\gamma h_2 + \gamma h_3(\delta d_{ij} + 1))} - 1 \qquad (5\text{-}16)$$

Coefficients were estimated by nonlinear least squares and are shown in table 12 along with the sample sizes.

Table 12. Estimated coefficients for the tree-within-plot height growth calibration models.

Species	m_{h_1}	m_{h_2}	m_{h_3}	No. Plots	No. Data Pairs
Redwood	1.49	1.30	-1.93	241	964
Douglas fir	1.21	2.06	-5.02	112	448

As height growth data is unavailable for the hardwood species, redwood coefficients are substituted for tanoak and Douglas fir coefficients are substituted for alder in model operation.

5.9.6 Tree-within-plot Variance Estimates

The simulation "startup" scheme described in chapter 6 requires estimates of the variances of the height and diameter tree-within-plot effects that are unexplained by the calibration functions. These are denoted are as $\sigma^2_{bd^*}$ and $\sigma^2_{bh^*}$ respectively. This section describes methods and procedures used to estimate these parameters.

5.9.6.1 Diameter growth models

Residuals from fitting (5-15) are inadequate in directly estimating $\sigma^2_{bd^*}$ as there is no way to partition the residual sums of squares into separate components. However, four successive measurements on a subsample of trees from the Jackson State Forest plot set were available for this purpose. Trees from this data set were selected if two or more height measurements were taken. Missing heights were estimated from a linear regression of height on elapsed time from plot establishment for each tree. Crown ratios were available on all occasions. On each plot, sampling proportions were constructed on the basis of numbers of trees in each crown class by species. Samples of trees with adequate measurements were then randomly drawn, based on these proportions, with the numbers of trees per plot being selected so as to produce the maximum number of trees and still maintain the sampling proportions. All analysis was species specific. The samples so drawn were then used to estimate vd_{ijk} producing a nested design of four measurements of J_i trees on I plots.

This set of variables was then analyzed three different ways to estimate a) σ^2_{bd} without any calibration functions; b) $\sigma^2_{bd^*}$ using coefficients estimated from (5-15); and c) $\sigma^2_{bd^*}$ with the coefficients γd_1 and γd_2 in (5-15) being estimated for each plot.

A comparison of the variances estimated with a) and b) above provide an indication of how well the general tree-within-plot calibration perform as the difference between these estimates is the amount of variation in tree-within-plot effects accounted for by the calibration function.

Conversely, a comparison between the variances estimated with b) and c) provides an indication of whether the general within-plot calibration function is a reliable predictor for specific plots.

a) No calibration functions

In this analysis, $\hat{vd}_{ijk}$ estimated with equation (5-11) was analyzed as a nested random effects model (Scheffe, 1959)

$$\hat{vd}_{ijk} = Ud + ad_i + bd_{ij} + \epsilon d_{ijk} \tag{5-17}$$

where Ud is the overall mean of the sample and is considered a fixed parameter. The remaining variables in (5-17) are treated as random.

b) General Calibration

This analysis was based on the following model

$$\hat{rd}_{ijk} = Ud + ad_i + bd^*_{ij} + \epsilon d_{ijk} \tag{5-18}$$

where

$$\hat{r}d_{ijk} = \hat{v}d_{ijk} - [\hat{\gamma}d_1 hd_{ij} + \hat{\gamma}d_2 cr_{ij}] \qquad (5\text{-}18a)$$

The average height/DBH ratio and crown ratio of trees for all four measurements were used as independent variables to access the general estimated tree-within-plot calibration functions (equation (5-15) with the coefficient estimates shown in table 11). (5-18) was similarly analyzed as a nested random effects model.

c) Plot-by-plot calibration

This analysis was based on the model

$$\hat{v}d_{ijk} = Ud + ad_i + \gamma d_{1_i} hd_{ij} + \gamma d_{2_i} cr_{ijk} + bd^*_{ij} + \epsilon d_{ijk} \qquad (5\text{-}19)$$

In this model, Ud and $\{\gamma d_{1_i}\}$, $\{\gamma d_{2_i}\}$ were treated as fixed parameters and the remaining variables as random. As with (5-18), the average height/DBH ratio and crown ratio of trees for all four measurements were used as independent variables. General theory for estimating variance components in mixed linear models of this type is summarized by Searle (1968) and adopted here to derive the mean squares necessary for subsequent estimation of variance components.

Variance components derived from these analyses are shown in table 13. Note that the estimates of $\sigma^2_{bd^*}$ obtained from analyzing (5-18) are 69% and 50% of the estimates of σ^2_{bd} obtained from analyzing (5-17) for redwood and Douglas fir respectively. These results indicate that the general tree-within-plot calibration functions account for a significant proportion of the within plot variation in tree effects. However, there is still a substantial portion of variation within plots that is unexplained. The comparable variance estimates obtained from analyzing (5-19) are not much different from (5-18) and would be an indication that relative differences between plots are minor.

Table 13. Variance estimates for tree-within-plot and combined time variant and measurement error components for three different diameter within plot calibration models by species.

Species	(5-13)		(5-14)	
	σ^2_{bd}	σ^2_{ϵ}	σ^2_{bd*}	σ^2_{ϵ}
Redwood	.251	.357	.173	.357
Douglas fir	.313	.237	.155	.237

unexplained tree var = .251 - .173 = .078 ∴ σ_{b*} = .28

Species	(5-15)		No. Plots	No. Trees
	σ^2_{bd*}	σ^2_{ϵ}		
Redwood	.145	.357	58	957
Douglas fir	.149	.237	29	342

Each plot is also capable of producing an estimate of the tree-within-plot variance component. It was suspected that the individual plot estimates of σ^2_{bd} would decrease in older stands as mortality tends to be concentrated in the "poorer" trees and, conceptually, tends to truncate the lower tail of the distribution of the tree-within-plot random variables. No significant correlations however could be found between the estimated tree-within-plot variances and average plot height. It was noted however that individual plot estimates of σ^2_{bd*} obtained from analyzing both (5-18) and (5-19) tended to be negatively correlated with average stand height. In older stands, the within-plot calibration functions similarly accounted for greater percentages of the total within-plot variation. Consequently, when estimates of $\sigma^2_{bd^*}$ are needed to develop equation modifiers for a single plot at the start of a simulation, the following estimate is used

$$\hat{\sigma}^2_{bd^*} = \hat{\sigma}^2_{bd} - MS_{bd_i} \tag{5-20}$$

where MS_{bd_i} is the mean square prediction of bd_{ij} for plot i obtained ~~using~~ using the tree-within-plot calibration function (5-15) and coefficients shown in table 11 and $\hat{\sigma}^2_{bd}$ is obtained from table 12. The net effect of this procedure is that in young stands, where the within-plot calibration functions account for a small proportion of the variation in bd_{ij}, $\hat{\sigma}^2_{bd^*}$ will tend to be "large". The converse is true in older stands.

5.9.6.2 <u>Height growth models</u>

As multiple height growth measurements on individual trees were unavailable, methods previously employed to estimate diameter growth tree-within-plot variance components could not be followed. Instead, the following indirect procedure was used which was dictated by data availability.

We initially assume that the largest 20% of trees by DBH in undisturbed evenaged stands also represent the upper 20% of the distribution of tree-within-plot height growth effects. Secondly, we assume this distribution is normal. Plotting of the estimates $\hat{t}h_{ij}$ indicated that this was a reasonable assumption for Douglas fir. For redwood, the individual growth plot distributions seemed somewhat skewed and a gamma distribution might be more representative but normality was assumed for practical purposes. The expected value of the largest 20% of the trees on plot "i" ($E[bh20_i]$) can be obtained from

$$E(bh20_i) = \left[\frac{1}{(.2)\sigma_{bh_i}\sqrt{2\pi}}\right] \int_{.84\sigma_{bh_i}}^{\infty} (bh_i)\exp\{(-1/2\sigma^2_{bh_i})(bh_i)^2\}\ \delta bh_i \qquad (5\text{-}21)$$

where .84 is the ordinate of a standard normal density function such that the probability of standard normal random variable being greater than .84 is .2. A sample based estimate of $E[bh20_i]$ was taken to be the average value of $\hat{th}_{ij}$ for the largest 20% of the trees by DBH on the plot. Denote this estimate as $(\widehat{bh20}_i)$. This estimate was substituted in (5-21) which was subsequently solved for an estimate of σ_{bh_i} giving

$$\hat{\sigma}_{bh_i} = .2\sqrt{2\pi}(\widehat{bh20}_i)/\exp\{-1/2(.84^2)\} \qquad (5\text{-}22)$$

Individual plot estimates were squared, weighted by the number of trees per plot, and averaged to form an estimate of σ^2_{bh}. This gave variance estimates of .036 for Douglas fir and and .063 for redwood. The estimate for Douglas fir was very close to that estimated by Mitchell (1975) for Douglas fir in the Northwest so it was concluded that the estimates were reasonable. As with the comparable diameter calibration analysis, it was suspected that the individual plot estimates of σ^2_{bh} would decrease in older stands, however no significant correlations could be found with average plot height. Hence, when estimates of the variance of tree effects unexplained by the tree-within-plot calibration functions ($\sigma^2_{bh^*}$) are needed for modifier assignment at the start of a simulation, the estimate of σ^2_{bh} is used in the same form of a relationship used in estimating $\sigma^2_{bd^*}$, i.e.,

$$\hat{\sigma}^2_{bh^*} = \hat{\sigma}^2_{bh} - MS_{bh_i} \tag{5-23}$$

where MS_{bh_i} is the mean square difference of the estimated $\hat{bh}_{ij}$ around the mean prediction for all trees in a plot tree list at the start of a simulation.

5.9.7 Commentary on Modifier Components

It is recognized that the development of the calibration functions and estimates of associated model system variances is somewhat *ad hoc* and suffers from objective rigor. Underlying reasons are attributable to the form and quality of available data. Subsequent evaluation however has indicated that the within-plot calibration functions and methods employed for determining $\hat{\sigma}^2_{bd^*}$ and $\hat{\sigma}^2_{bh^*}$ usually perform adequately in model operation. In some situations though, inconsistencies may arise. It was noted that in young stands, the variation in the height-diameter ratio and crown ratio within plots tends to be relatively small as size differentiation has not had much time to occur. In these situations, the unexplained variation in tree effects is assumed to be large as little is explained by the calibration functions. However, in older stands, it is quite possible for a thinning operation to leave trees of such uniform nature that the mean square prediction of the calibration functions will also be small. In this case, the methods also employed assume a large value for $\sigma^2_{bd^*}$ and as such is probably an overestimate. The situations where these inconsistencies arise essentially make use of more information than is recognized in the model.

As noted earlier, the plot calibration functions do little to reduce the between-plot source of variation. It was noted during the course of this analysis that use of the average height to DBH ratio and average crown ratio, as additional explanatory variables in the plot calibration functions, could account for about 40% of the variation in plot effects. In some cases, these variables proved to be effective additions to the predictive model (equation 5-14). In stands where most of the overstory had been removed, average height to DBH ratio increased and predictions indicated that the leave trees are relatively "poorer" growing ones. In these situations, the predicted plot effects would be in accordance with what is probably happening in the woods. In younger stands however, average height over diameter ratios increase with increasing density without much apparent change in average crown ratios. Apparently, competition has an immediate impact on diameter growth while it takes much longer before it has an appreciable impact on crown recession. There is no reasonable basis for assuming that plot effects should be different for stands of different densities. In view of this inconsistency, the simple plot calibration functions previously described were retained as component models of the system. As a consequence, estimates of plot effects for initializing the model system tend to be overpredicted for stands that have been subjected to overstory removals in the past. Conversely, underpredictions are usually obtained for stands that have been thinned from below. These situations however can be remedied by using actual past growth measurements to effect a local calibration. Possible procedures for this are discussed in Chapter 7.

5.10 Summary

In the preceding analyses, explicit model forms have been derived and the relevant parameters needed to implement the the model system have been estimated. This system has been pieced together from several data sources using data that is less than "perfect" and estimation procedures, that, while reducing some analytical problems, are essentially *ad hoc* without clearly identifiable sampling properties. As a consequence, total internal compatibility of all the component models has not been insured.

It is emphasized that the analysis presented here is not an initial and unchecked attempt. Rather, it is a third or fourth generation result that has evolved from implementing a model system, performing numerous operational tests, identifying possible problems, reformulating the design, trying alternative methods of quantifying relevant relationships and repeating the process. While the procedure is conceptually neverending, further refinement of the current state of the model system is hampered by lack of appropriate data. However, as indicated in chapter 5, the system seems to perform reasonably.

Chapter 6

MODEL IMPLEMENTATION AND EVALUATION

The previous description of the model system and resulting parameter estimates provides the necessary material for implementing the tree growth system. This model has been coded into an interactive computer program called CRYPTOS[1/] that accepts the tree list and other data described in chapter 2 and modifies the tree list for growth or harvest operations at the user's direction. This allows users to simulate partial harvest and to tabulate conventional forest growth and inventory summary statistics. The actual computer program is documented in more detail elsewhere (Krumland and Wensel, 1980a). Figure 9 shows the conceptual flow of operations and functions of the the simulation program.

The remainder of this chapter describes the simulation initialization strategy and an evaluation of model performance.

6.1 Simulation Initialization Strategy

As noted in chapter 2, incorporation of random variation in the model is one factor that is used to produce stand diameter distributions that are characteristic of coastal stands after several decades of simulations. Being able to predict future diameter distributions is a desirable model attribute because many of the items of managerial interest are based on nonlinear functions (i.e. tree volumes) of tree characteristics as well as sums of these functions stratified by tree size classes (i.e., tree and log size stock tables). It follows that a

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1/ CRYPTOS is an acronym for Cooperative Redwood Yield Project Timber Output Simulator.

CRYPTOS Program Environment

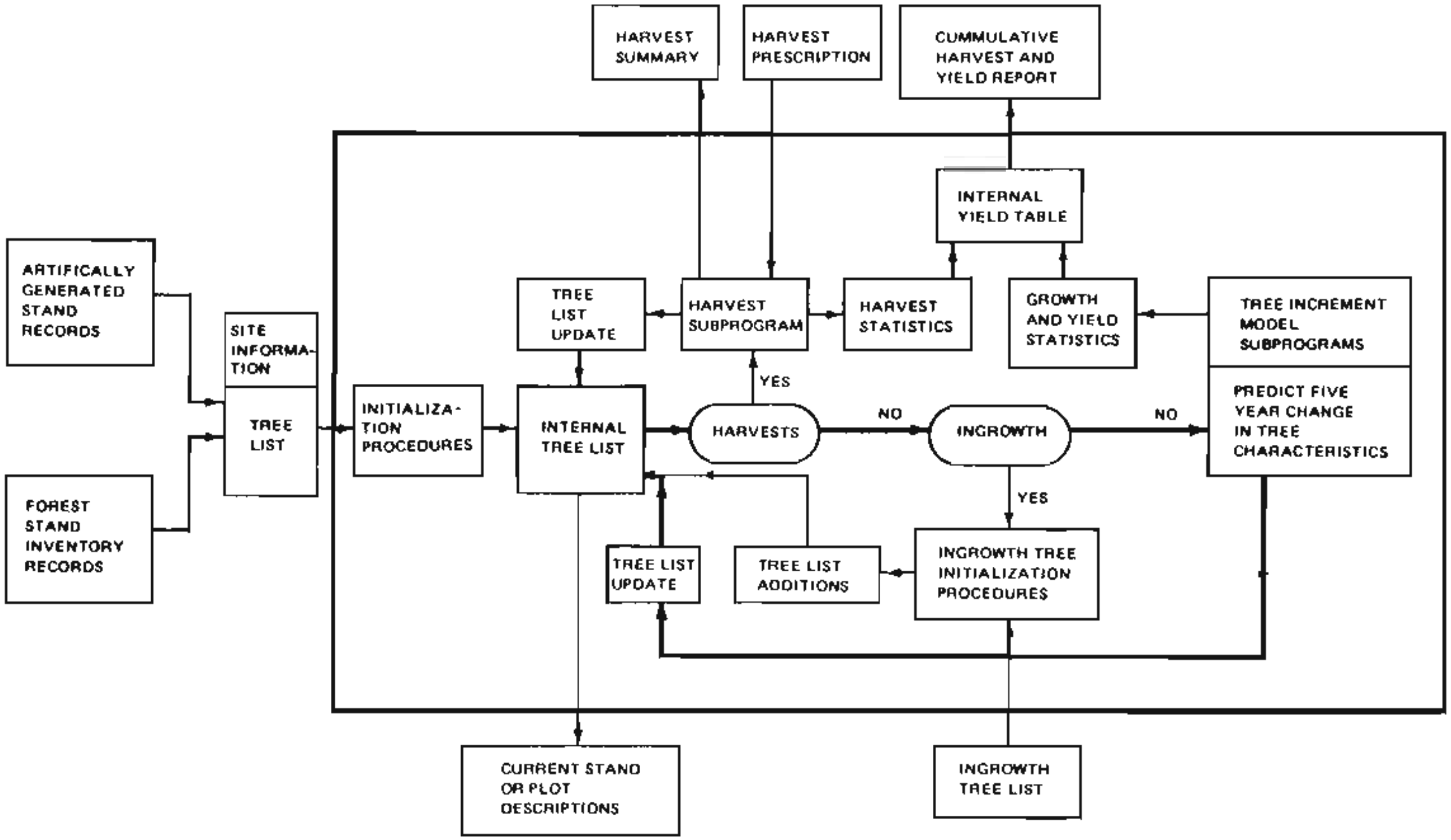

Figure 9. CONCEPTUAL FLOW CHART INDICATING GROWTH MODEL LINKAGES WITHIN THE CRYPTOS PROGRAM

suitable scheme for incorporating random variation in tree growth is an important feature of the model system.

The initial steps involved in initializing the computer program that contains the model system are summarized as follows:

a) Site index information and a list of tree characteristics for a specific plot as described in chapter 2 are read into the program.

b) This tree and plot information is then used to to compute the independent variables necessary to access the plot and tree calibration functions described in chapter 5.

c) Each tree is then given a predicted plot and tree, height and diameter growth random effect ($\hat{a}d_i$, $\hat{a}h_i$, $\hat{b}d_{ij}$, $\hat{b}h_{ij}$) where the two plot effects are the same for all trees of a given species.

The predicted plot and tree effects account for some of the time invariant variation in tree growth. However, as noted in chapter 5, some variation is still unaccounted for which is to be introduced as additional stochastic effects in model operation.

6.1.1 Introduction of Random Effects

There are several possible ways to incorporate unexplained variation in tree growth in simulations. One avenue to pursue would involve a scheme based on several simulations, where at each trial, an assignment of the unexplained random components in the model (plot, tree, and time variant random effects) is made for each tree based on a random drawing from appropriate probability density functions. Since a primary forecasting goal is to produce the average result of a random process, this procedure would have to be repeated several times. Unless the purpose of the forecast is to assess the variability of the outcome of the simulation, this procedure has several drawbacks; a) it is tedious and time consuming and b) it is not replicable. Nonreplicability may unduly compound comparisons made of the same stand under different simulated harvest regimes.

As a consequence, the following procedure and conventions have been adopted to accomplish two objectives; a) only one simulation is necessary to produce results that have the appearance of averaging several simulations with random components, and b) the simulation is replicable for identical initial tree lists and management scenarios[2/]:

1) Possible variation between plots is not considered in the stochastic scheme. Hence, the relative level of the whole plot is determined soley by the plot calibration equations. Similarly, for reasons discussed in chapter 2, time variant sources of variation are not considered.

2) The distributions of the "unexplained" tree effects, bd_{ij}^* and bh_{ij}^* are assumed to be normal and independent of each other.

3) The corresponding normal probability density function associated with each unexplained tree effect is then partitioned into three discrete fractions representing the lower 25%, the middle 50%, and the upper 25% of the cumulative distribution. For each tree record, the means of these fractions are obtained by integration of the normal probability density function. Hence, it is assumed that the distribution of tree effects can be approximated by a discrete probability space with three outcomes. As an example, the ij^{th} tree record has a diameter growth tree-within-plot random effect that can take on the following outcomes with the indicated

2. This procedure is a variant of a simulation scheme described by Stage (1973).

probabilities

location	assigned mean	probability
lower 25%	$\hat{b}d_{ij} - 1.16\hat{\sigma}_{bd_i^*}$	.25
middle 50%	$\hat{b}d_{ij}$	.50
upper 25%	$\hat{b}d_{ij} + 1.16\hat{\sigma}_{bd_i^*}$	.25

Where $\hat{\sigma}_{bd_i^*}$ is determined from (5-20). A similar procedure produces analogous counterparts for the height growth tree effects. This procedure then gives nine possible combinations of height and diameter growth tree effects with the probability of any pair being the product of the respective marginal probabilities. A procedure initially adopted involved a) replicating each tree record eight times giving nine identical records, b) assigning each of the nine possible pairs of tree effects to a replicated tree record, and c) multiplying the per acre weight of the original tree record by the probability associated with each pair of tree effects and assigning it to the corresponding replicated record.

The overall impact of this procedure is that the contribution of the original tree record to the plot inventory is unchanged and the average of both tree effects are still equal to the pre-replication levels. For either height or diameter growth, the respective tree effect assigned by the previous procedure is added to one plus the predicted plot effect. This produces the increment equation modifier for the tree record which operates as a multiplicative adjustment to predicted growth.

Nine replications of the original number of tree records however, resulted in a substantial decrease in real time simulation efficiency on an interactive PDP 11/70 computer. Consequently, four of the nine pairs of possible replicates were chosen as indicated in table 14. This reduced scheme still maintains the same marginal probabilities as the nine-pair scheme and the mean tree effects for each of the four replicates is still the same as the original record. Each of these four pairs also has the same joint probability so the original per-acre weight is multiplied by .25 and assigned to each of the four associated records.

Table 14. Pairs of unexplained height and dameter growth tree effects selected for use (x) in simulation initialization.

		Height Growth			
	Location	Upper 25%	Middle 50%	Lower 25%	Relative Mean
	Upper 25%		X		$1.16\hat{\sigma}_{bd}$*
Diameter Growth	Middle 50%	X		X	0
	Lower 25%		X		$-1.16\hat{\sigma}_{bd}$*
	Relative Mean	$1.16\hat{\sigma}_{bh}$*	0	$-1.16\hat{\sigma}_{bh}$*	

6.2 Evaluation of Model Performance

Evaluation of model performance has been approached from three viewpoints: 1) conformance of model predictions with generally accepted tenets of forest growth; 2) comparisons with other models; and 3) comparisons with actual forest growth plot development.

6.3 Conformance of Model Predictability

To facilitate evaluation of the model, a tree list generation model coded into a computer program was utilized to create representative tree lists for evenaged stands with specified characteristics. As input, this model requires, for each species, a site index, breast high age of dominant trees, numbers of trees per acre, number of tree records to be generated, and the DBH of a tree of average basal area. (This latter characteristic is optional and the program will supply an estimate if it is not provided.) The stand generation model is documented in more detail elsewhere (Krumland and Wensel, 1980b).

The CRYPTOS program has been used to project growth and yield for generated stands composed of several combinations of site index, stems per acre, and species mixes. The general impression gained during this process was that the simulations looked relatively reasonable. Periodic stand basal area growth monotonically decreases as a function of age although for very sparse initial conditions or young stand ages, growth sometimes initially increased and then decreased. Periodic cubic foot growth reached maximums at ages later than basal area growth and ages earlier than board foot growth. The peak growth was at younger ages for the higher sites and the denser initial stocking levels. These observations are in general conformance with existing precepts on even-aged stand development and in general agreement with several studies of even-aged stand development (cf. Vuokila, 1965).

Differences between individual species, however, are quite noticable. Figure 10 shows basal area growth curves simulated for pure even aged stands of each of the four main species components of the model system. Initial starting states for each stand were 300 stems per acre at 10 years of age. Site indices for each species were specified to be approximately the average site index of the particular species in the available sample growth plots. The growth of Douglas fir and tanoak tends to be somewhat "classical" in curve shape. The prediction for alder, while being based on little more than informed judgment, indicates the stand begins to deteriorate at about seventy years of age. Several field foresters in the region have indicated that alder stands do in fact begin to break up at around this age. Redwood on the other-hand, maintains a relatively high level of basal area increment throughout the entire age range usually associated with young growth stands. No known yield models of any other conifer species indicates this type of growth behavior. However, redwood isn't like any other species of conifer and exhaustive comparisons with actual growth records confirms the general level of predictions shown here.

How individual species interact with each other is an important management concern, particularly when contemplating harvest regimes that may be species specific. Figure 11 and 12 are examples of simulated dynamics of a stand initially composed of 100 stems per acre at age ten of each of the four main species in the model system. Site indices have been specified to represent the species differentials that commonly occur in coastal stands. While a stand with this composition may be rare (alder is usually found on wet sites and tanoak on drier ones),

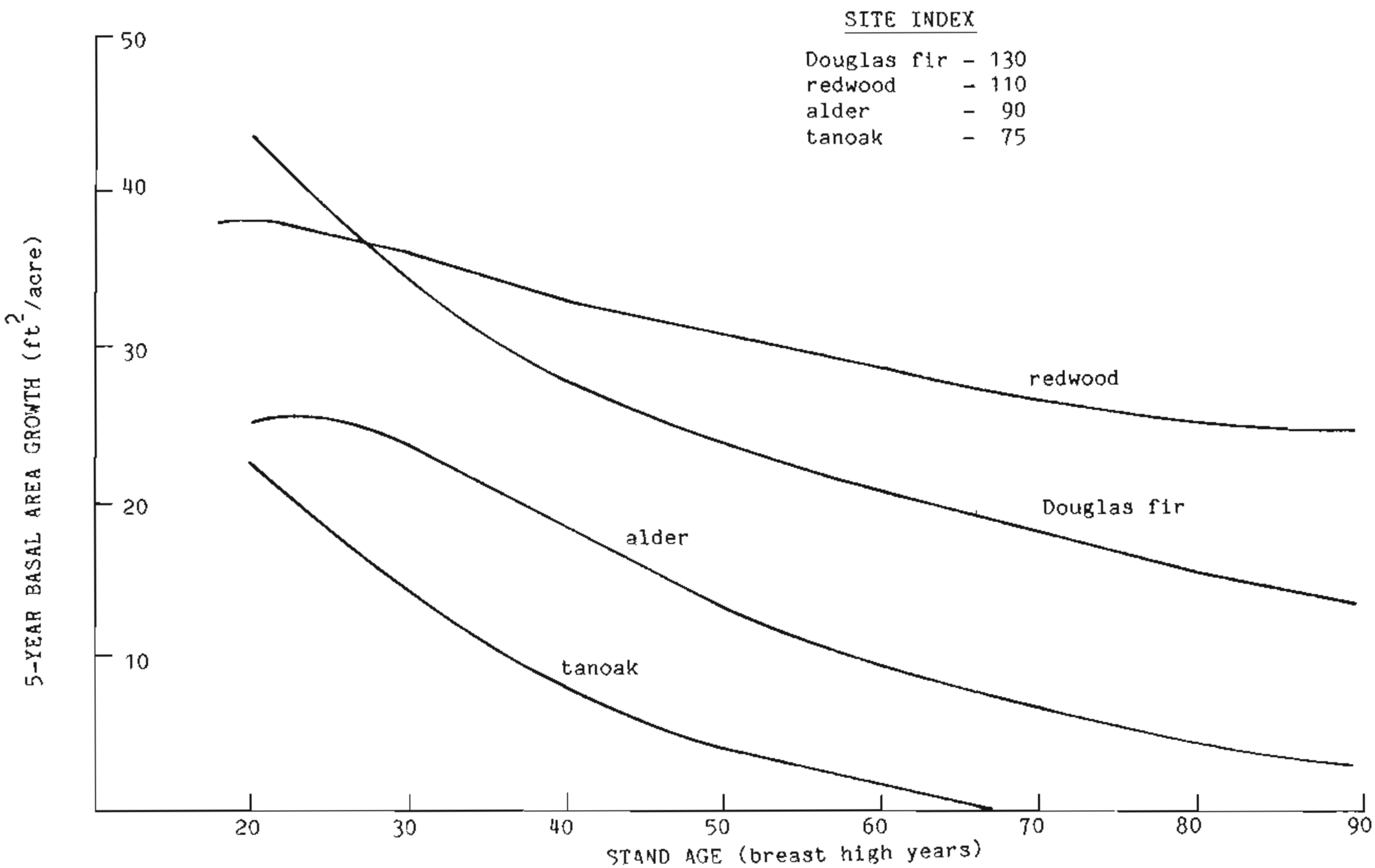

Figure 10. Simulated basal area growth for individual pure species stands composed of 300 stems per acre at age 20.

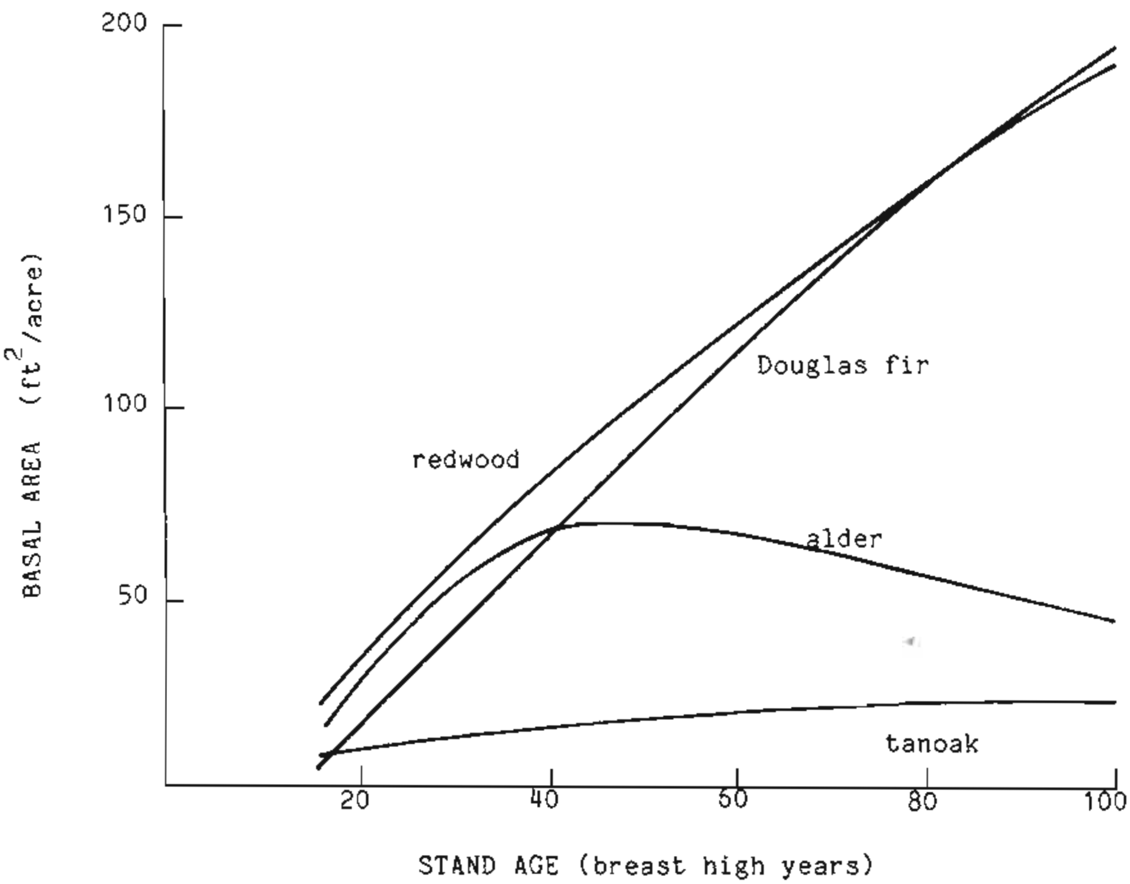

Figure 11. Basal area yields of a mixed species stand composed of 100 stems per acre of each species at age 10.

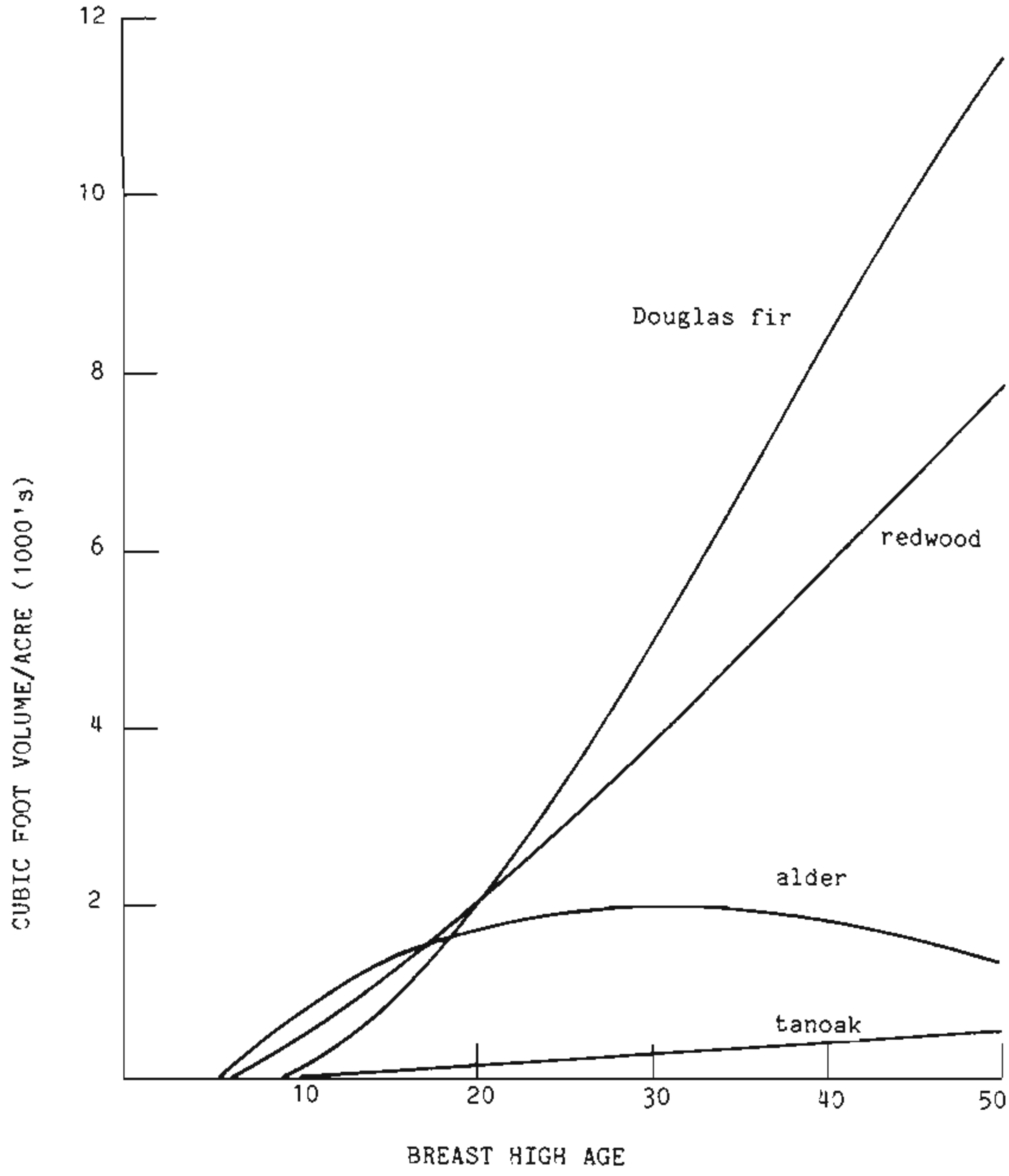

Figure 12. Cubic foot volume yields of a mixed species stand composed of 100 stems per acre of each species at age 10.

these figures demonstrate some general attributes of the model; rapid initial growth of hardwoods and later becoming overtopped and suppressed by the conifers; early domination of the stand by redwoods; and Douglas fir gradually catching up with redwood and becoming the dominant stand component later in the life of the stand.

As an indication of the effect of the stochastic features on model performance, a ten year old pure redwood stand was generated and grown for 70 years in conjunction with the stochastic scheme previously described. This simulation was then repeated with all tree effects set to zero. Thus, the modifiers for all trees were the same in this trial. The resulting stand diameter distributions for both of these trials are shown in figure 13. The diameter distribution resulting from simulating with the stochastic scheme has a wider spread and is less symmetric than what results from simulating without stochastic features. As an alternative method of describing diameter distributions, Krumland and Wensel (1979) have developed a model that predicts parameters of stand diameter distributions as a function of current stems per acre, average stand diameter, and stand age. To compare the predicted distribution resulting from this model with the ones resulting from the two simulations, the average terminal stand summary statistics at age 80 of both simulation trials (trees per acre and average stand diameter) were then used as access variables. The predicted distribution is also graphed in figure 13. If this predicted distribution is used as a representative measure of what stands of these characteristics should look like, the stochastic scheme adopted for model operation would seem to be performing reasonably. Conversely, no stochastic effects at all in model operation produces an unrepresentative shape and a distorted

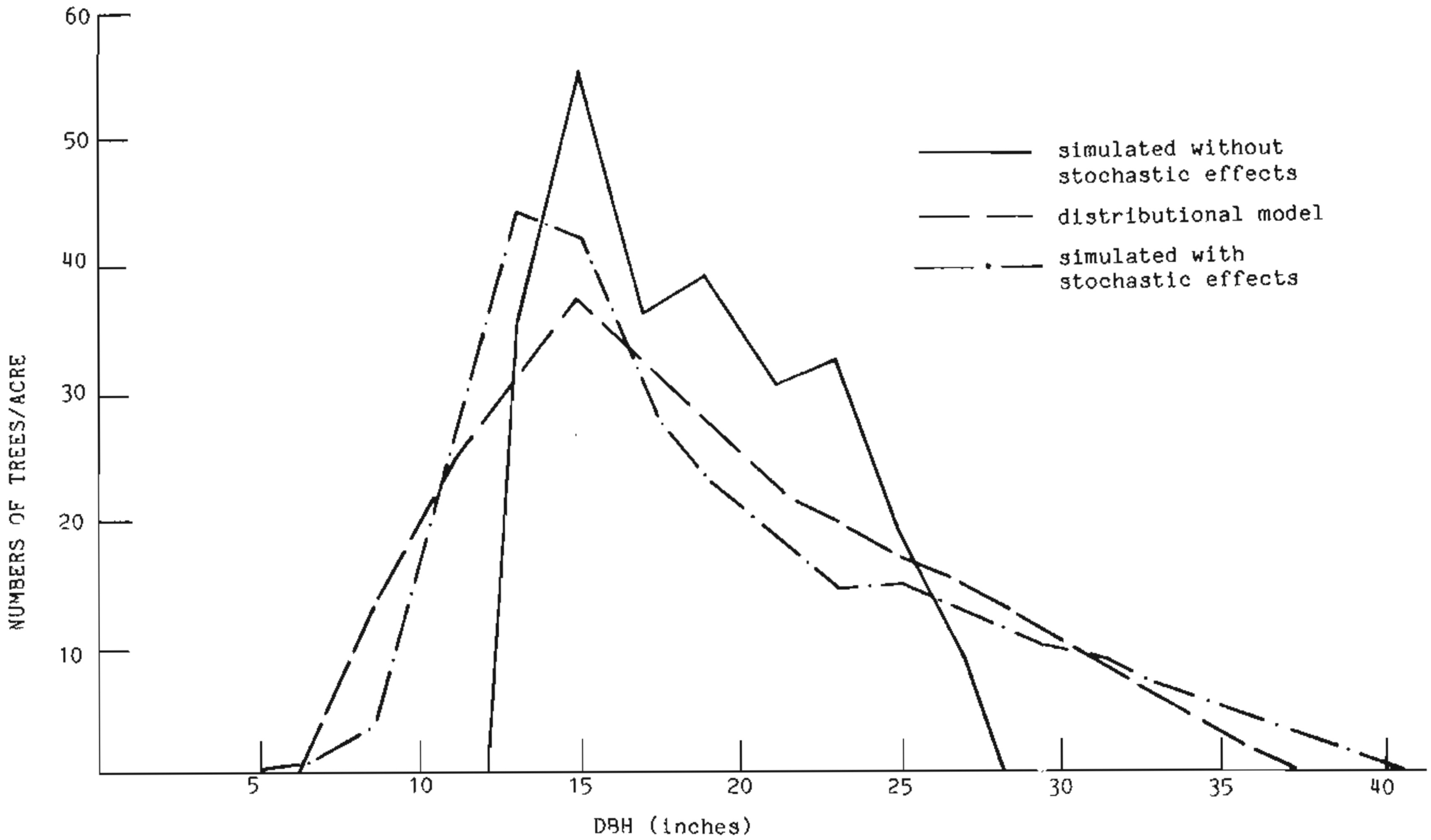

Figure 13. Redwood diameter distributions resulting after 70 years of simulation with and without stochastic effects compared to a diameter distribution model.

prediction of numbers of trees by size classes.

6.4 Comparisons with Other Models

Comparisons with other models are not necessarily definitive as differences inevitably lead to the question of which one is "correct". However, if different models, employing different methodologies and data sources, purport to describe the same phenomena, similarities can be viewed as an indication of model reasonableness.

6.4.1 Model Comparisons for Douglas fir Basal Area Growth

There are no growth studies for Douglas fir in coastal California with the exception of the "normal" yield tables prepared by Schumacher in 1949. These were not considered applicable.[2/] Chambers (1980), however has developed a growth model for Douglas fir in Washington. This model predicts annual stand growth in basal area as a function of current basal area, age, and site index. Hence, it is a "type B" model used recursively in a "type A" situation and subject to problems previously discussed. It is, however, the best comparative model readily available. Wiley and Murray (1974) have also developed a whole stand Douglas fir model similar in function to the one Chambers has developed except the data they used were restricted to stands less than 35 years in age. Comparisons were made with both of these models.

[2/] The traditional normal and empirical yield table were constructed by stratifying measured inventory plot attributes such as average DBH, basal area, etc. of a somewhat subjectively chosen sample on the basis of site index and age. No growth measurements were used in construction although the somewhat suspect practice of using the difference in inventory estimates of different stands at different ages as an estimate growth is frequently done in practice.

Several comparisons were made based on generated tree lists representing pure Douglas fir stands of different initial stems per acre and site indices at age ten. The basal area from the initial tree list was used as a starting basis for for Wiley and Murray's model. Predicted stand basal areas at age 35 resulting from this model were used as a starting basis for Chamber's model because he based his model on trees larger than seven inches in DBH and at age 35, most of the trees in typical Douglas fir stands are larger than this minimum.

One of these comparisons for a well stocked stand is shown graphically in figure 14. In this and in other comparisons, there is a general similarity in growth trajectories but the projections made by the tree model (CRYPTOS) are consistently 10-20% higher than the others.

Such uniform differences would tend to discount the possibility that the discrepancies are due to differences in methods and model forms (stand versus tree based). Differences in yields between Douglas fir in California and in Washington and Oregon have also been noted by Schumacher (1930). He compared basal area and numbers of trees per acre of the sample plots he used in developing the normal yield tables for California Douglas fir with the tabled values of normal yields for Douglas fir in the Northwest. Trees per acre averaged 4.5 percent less for California Douglas fir stands but basal area was 33.2% higher. Thus it seems that there are real differences in growth and yield between northern and southern Douglas fir.

6.4.2 Model Comparisons for Redwood Basal Area Growth

The most recent redwood growth studies are the empirical yield tables of Lindquist and Palley (1963) and the companion study for prediction of ten-year stand increment (Lindquist and Palley, 1967). The

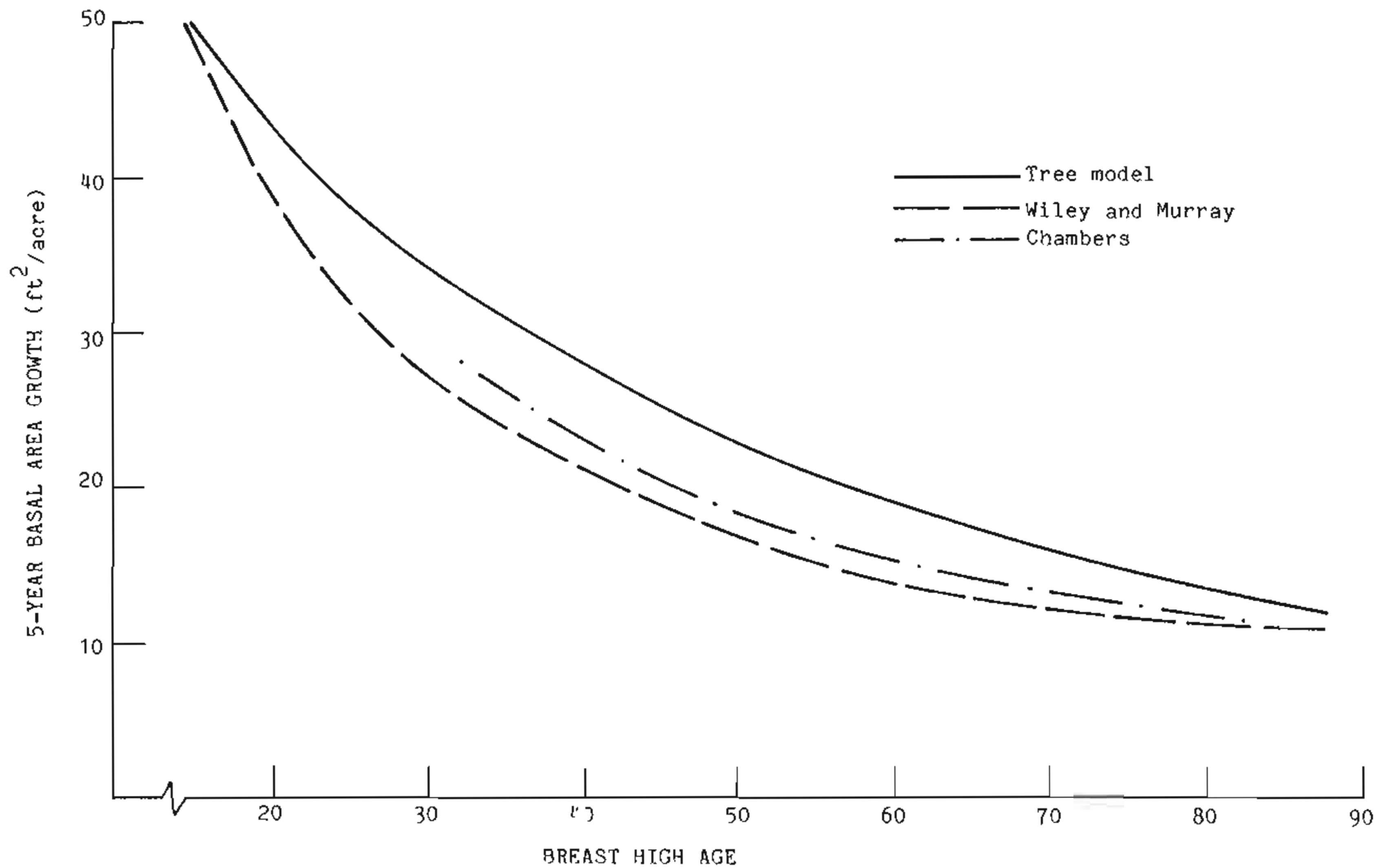

Figure 14. Model comparisons of Douglas fir basal area growth. Initial conditions at age 10: 400 trees and 38 square feet of basal area per acre.

empirical yield models were constructed by traditional yield table methods and may not be representative of actual growth patterns. The same data set, however, had ten year growth measurements and was used to make a "type B" whole-stand model for basal area increment. The ten-year growth model predicts 10-year growth in basal area as a function of current basal area and age. Consequently, it can be used recursively in a "type A" situation to make long term predictions. As hypothesized by the author (cf. Chapter 3), however, this may result in overpredictions. One factor that may tend to reduce the usefulness of any comparisons results from the Lindquist and Palley model being based on stands that were not pure redwood. By basal area, their sample plots averaged 84% redwood, 14% other conifers, and 3% hardwoods. The distribution of species by stand ages is not known. One noticable aspect of mixed-species stands that became evident in simulation trials, and has been substantiated by actual growth data, is that the species composition of stands by basal area is not stable over time. In young mixed-species stands, a usual pattern is that redwood predominates in terms of basal area. However, as stands get older, the relative proportion of basal area in Douglas fir tends to increase. Lindquist and Palleyi, being only concerned with short term growth estimates, did not attempt to model long term changes in species composition. Hence, a five year basal area growth model was developed from the plots available to this study that were 95% or greater by redwood in basal area. This sample restriction reduces the compexities of species dynamics and allows a more compatible redwood comparison. The form of this model was the same as the Lindquist and Palley model. The two stand models and the tree-based system were then used to simulate basal area development for comparable initial conditions.

The results of one set of comparisons that is typical of the differences between these three models is shown graphically in figure 15. There is general agreement in level between the stand model developed in this study and the tree based system. The general difference in the level and trajectory of the growth projections of the Lindquist and Palley model may be due to several factors. First, as mentioned earlier, their data is based on stands that are not pure redwood. Simulated basal area growth of mixed redwood and Douglas fir stands with the tree based system indicates trajectories that are more in conformance with the Lindquist and Palley model. Second, the sample basis of the Lindquist and Palley study was restricted to the "better" stocked stand conditions where as the sample basis for the other two models did not use such a restriction. Finally, the data Lindquist and Palley used were taken from the period 1947-1957 whereas the data used in this study was largely collected after 1960. As indicated later in this chapter, there may be significant differences in growth associated with different calendar periods.

In any event, while differences were observed between the growth rates shown in this study and the Lindquist and Palley study, the redwood basal area increment component of the system seems to be performing adequately.

6.4.3 Model Comparisons for Tanoak Basal Area Growth

Tanoak growth studies are rare as the species is usually considered by industrial foresters to be a "nuisance" rather than an item of commercial importance. Nguyen (1979), however, has fit a stand-based yield projection model described by Sullivan and Clutter (1972) to 24 sample

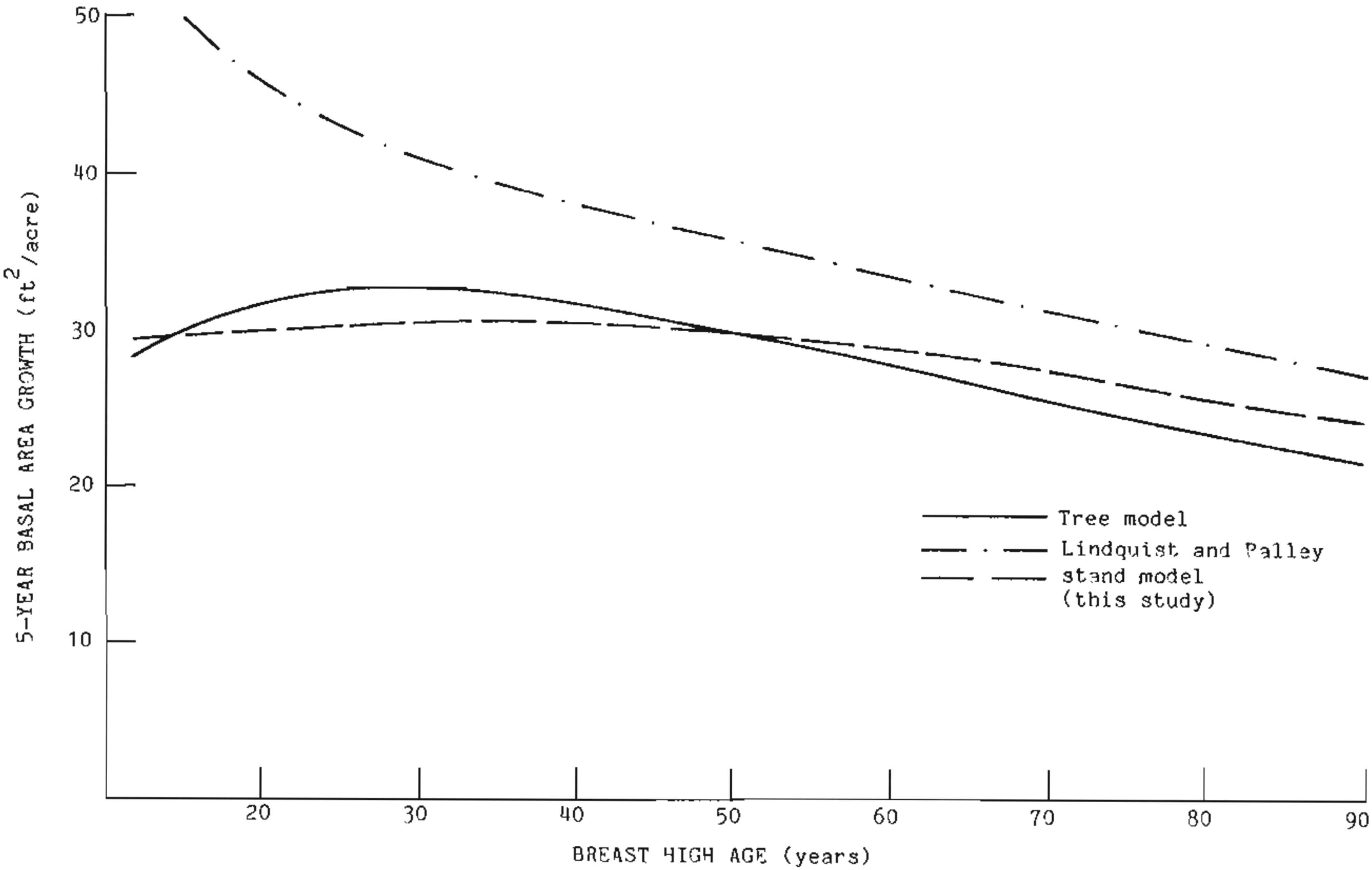

Figure 15. Model comparisons for redwood basal area growth. Initial conditions at age 10: 250 trees and 41 square feet of basal area per acre.

plots located in relatively pure tanoak stands in Humboldt county. The age of tanoak on these plots ranged from about 15 to 50 years. Some of these plots were also used in the present study. Cumulative basal area growth curves predicted by Nguyen are shown in figure 16 along with the predictions made by the tree model system. Differences between these two prediction methods are negligable throughout the range of conditions likely to be encountered in tanoak stands so it was concluded that without any other forms of comparitive evidence, the tanoak component of the model system was reasonable.

6.4.4 Height Growth Comparisons

The performance of the height growth models was considered to be crucial to system operation because the three increment models all used total height or current estimates of height growth as explanatory variables. Coupled with the sometimes imprecise measurements used in in model development, some extensive comparisons of model performance were considered to be in order.

The site index models which formed an integral part of the height growth models provide stable and representative estimates of height growth for dominant trees. Quantitatively, the site index tree component of a stand is the stand fraction delimited by the upper 20^{th} percentile of the stand DBH distribution for a particular species. Extensive simulations were made for individual species, where for several initial levels of stems per acre and site index values, a ten year old stand was generated and growth was simulated up to age one hundred. At each ten years of elapsed time in simulations, the average height of the largest twenty percent of trees by DBH was computed and compared with the estimate from the site index equation. The largest differences

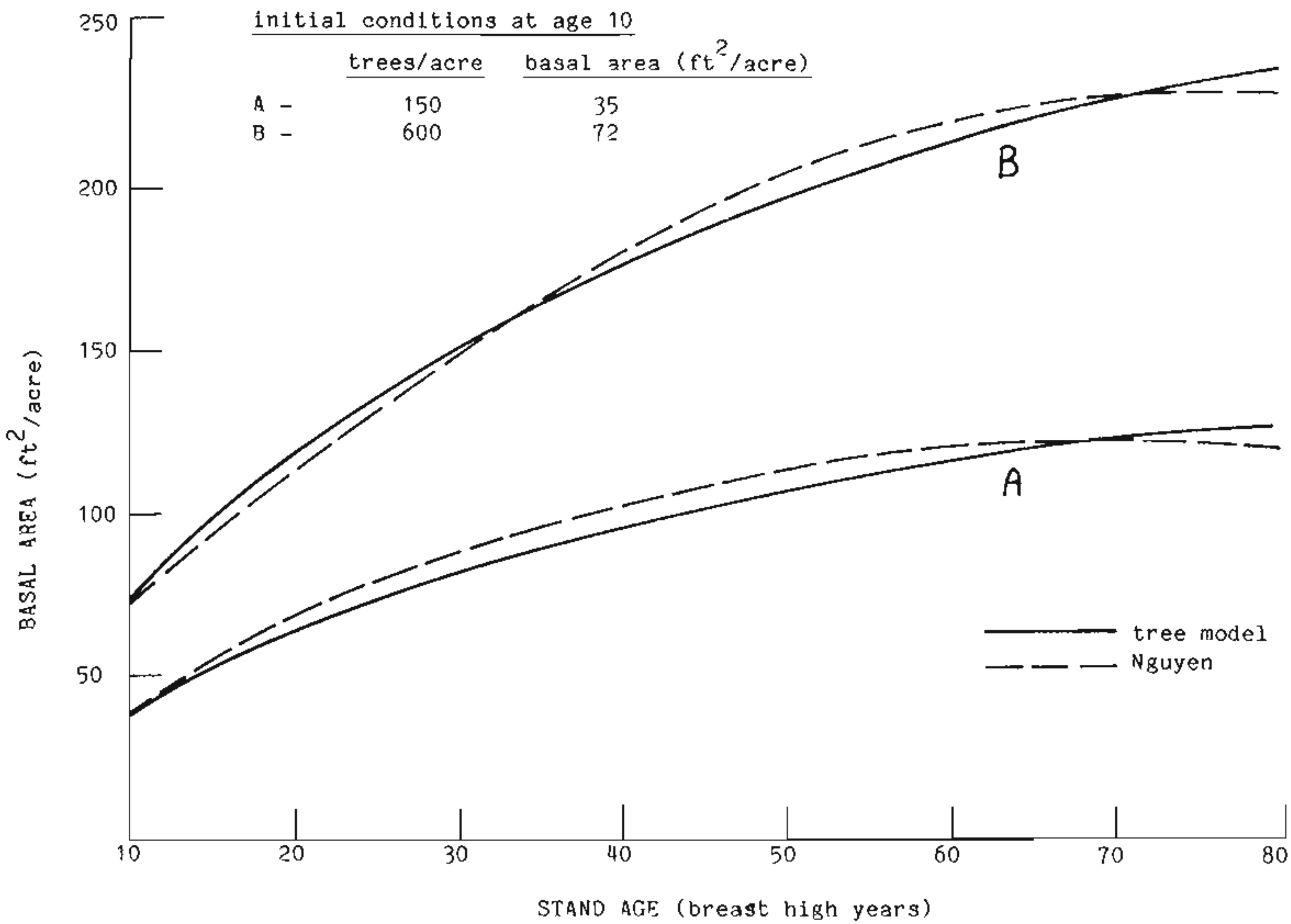

Figure 16. Model comparisons of tanoak basal area yields.

occurred at the oldest simulated age and were almost always less than ten feet. As typical dominant 100 year old conifers tend to be 140-180 feet tall, differences of this magnitude seemed acceptable. Conformance between simulated dominant height and site curves is also an indication that the entire calibration system and the stochastic scheme are functioning adequately as the comparitive component represents trees in the upper tail of the distribution of the within-plot tree effects.

The only external means of comparing height growth of stand fractions other than dominants is the model that predicts tree heights used in the stand generation model. This model is described by Krumland and Wensel (1978a) and predicts tree height (h_i) given the average height of dominants (HDS), average DBH of dominants (DDS), and tree DBH. This model was found to be quite precise in estimating a conditional height distribution. Several simulations were performed where stands of different initial stems per acre were generated at age 10 and growth subsequently simulated to age 80. Average heights simulated by the model were then computed for two-inch DBH classes and graphically compared to predictions made with the height estimation model using independent variables computed from the simulated 80 year old stand. Overall, there was a general conformance between simulated and predicted height distributions. Figures 17 and 18 show comparitive results for sparse and heavily stocked initial stand conditions for redwood and Douglas fir respectively. Differences between the two simulated height distributions results from a greater amount of competition and hence, less growth for the smaller trees in the heavily stocked conditions.

In summary, the fact that simulations over seventy years produce height distributions that we see in stands today, supports the

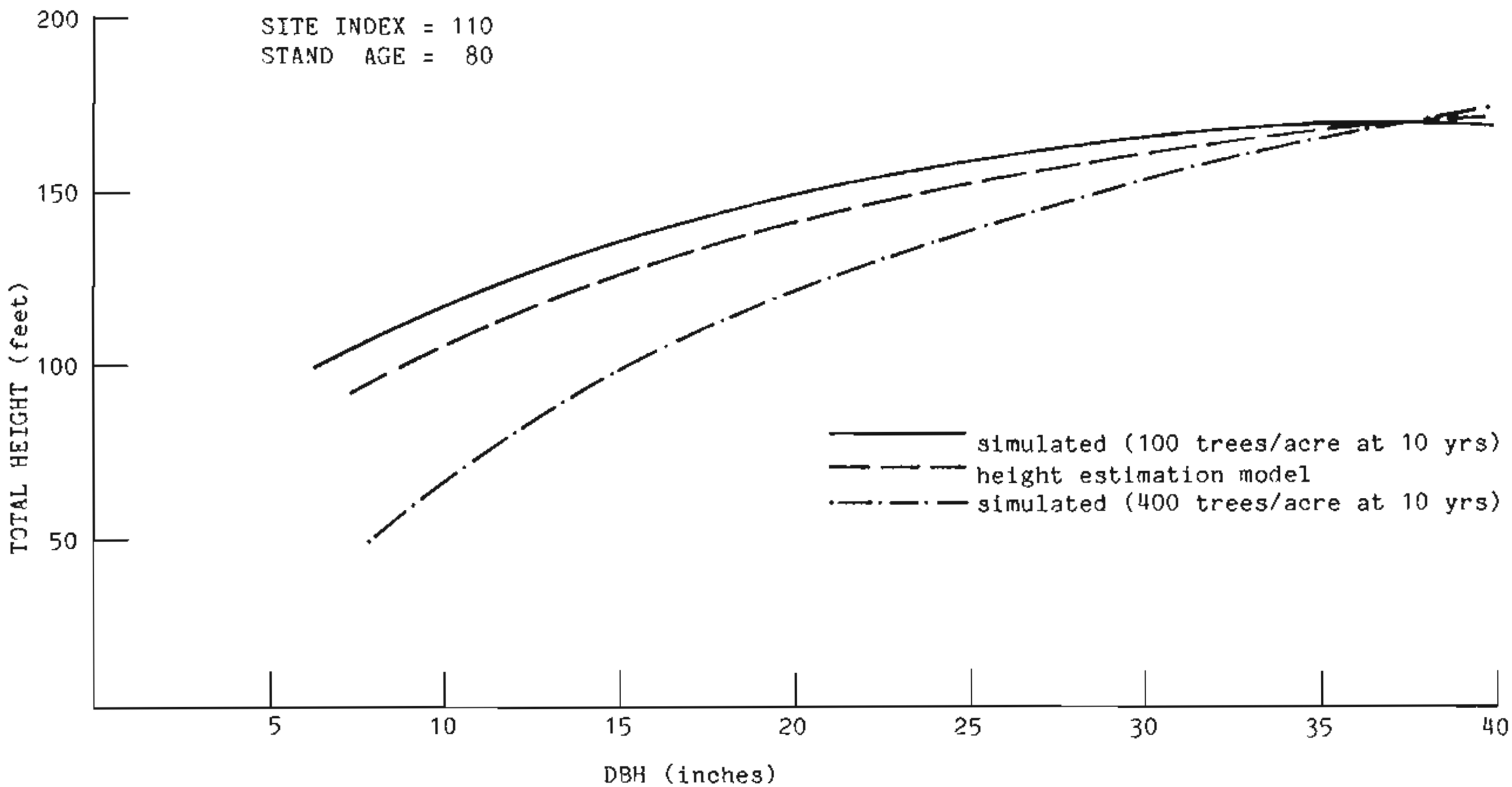

Figure 17. Average height by 2-inch DBH classes for redwood after 70 years of simulation for two different initial stocking levels compared to a height estimation model.

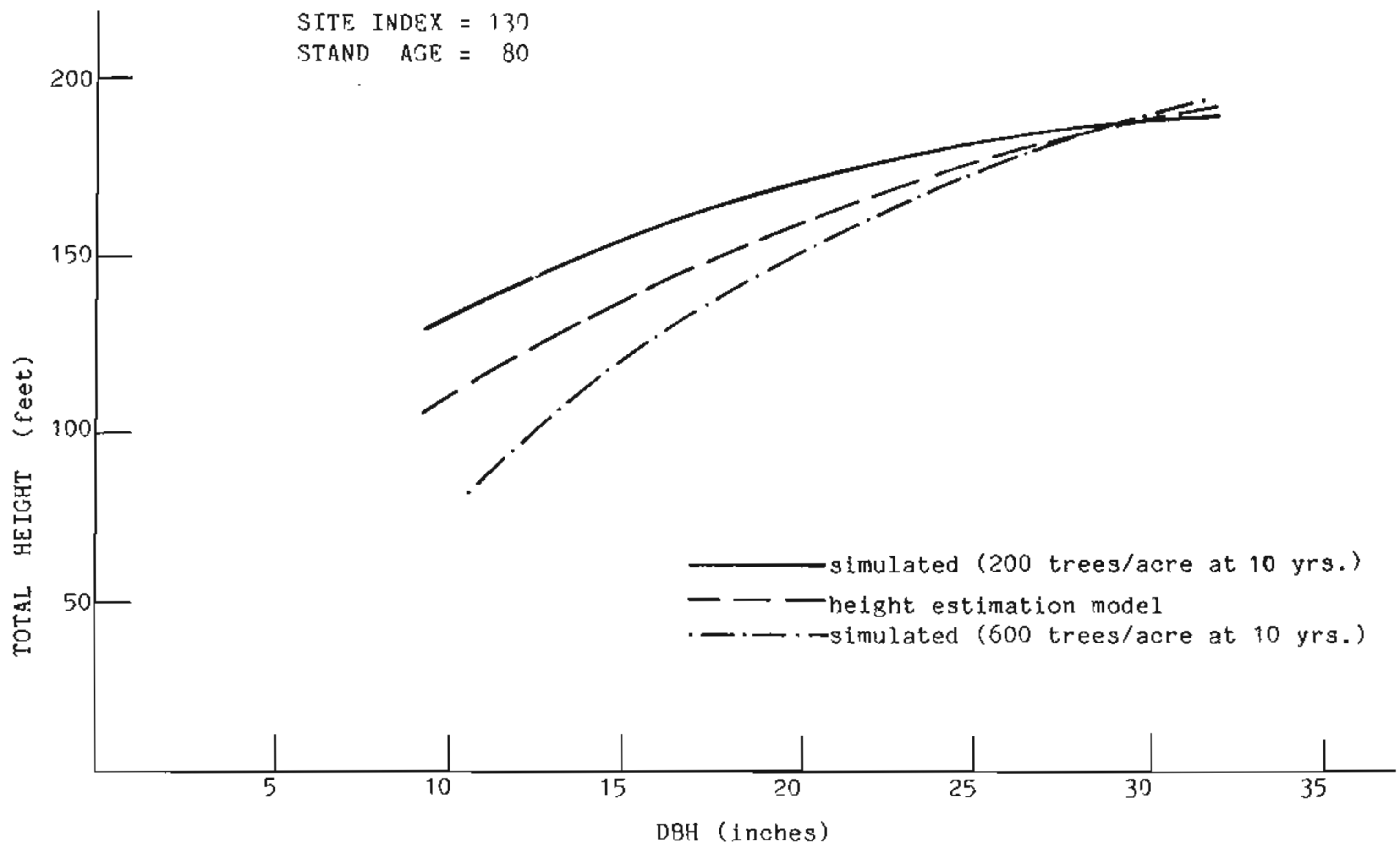

Figure 18. Average height by 2-inch DBH classes for Douglas fir after 70 years of simulation for two different initial stocking levels compared to a height estimation model

hypothesis that the basic height growth trajectory of the model is correct.

6.4.5 Possible Problems

Comparisons similar to the ones previously described were also made for the crown recession models. Usually, throughout the range of typical young growth stand conditions, this component followed the pattern expected. However, in very old stands or extremely dense stand conditions, rates of crown recession sometime seem to be overpredicted. The impact on basal area increment, however, is not much effected as an underprediction of tree crown ratio (negative effect on growth) is compensated by a an underprediction of canopy cover percent (positive impact on growth). The problem, however, warrants additional attention if better data ever become available.

6.5 Comparisons of Model Performance with Plot Growth

Ultimately, growth models have to be capable of making accurate and consistent predictions of stand performance before they can be generally accepted as useful forest management tools. While accuracy standards are subjective, comparisons of simulated growth with actual plot development records can be used as evidence in testing the validity of the model.

The comparisons made in this section use plot basal area growth as the primary variable of interest. Tree heights and crown sizes on comparison plots were either totally absent at the initial measurement or confined to a subsample. Lack of total height measurements effectively eliminates any refined comparisons of volume growth. In order to initiate the simulations, the missing measurements had to be estimated using models from the stand generation program if no measurements were taken at all, or from local plot based models if a subsample was available. As a consequence, differences between actual and simulated plot performance may be exaggerated.

6.5.1 Union Lumber Company Plots

In 1952, personnel of the former Union Lumber Company established several one-fifth-acre growth plots in typical evenaged mixed redwood-Douglas fir young growth stands.[3/] These plots had been placed in moderate to well stocked stands and located in relatively homogenous areas with adequate buffers as protection from possible external influences. The trees on these plots had been bored for past 10-year radial increment thus allowing a reconstruction of the plot DBH measurements to 1942. Subsequently, these plots had been subjected to sporadic and inconsistent measurements. In the summer of 1976, 12 of these plots were located and remeasured. A minor amount of ingrowth was evident on some plots which was ignored in the comparisons. Thus, a 34 year measurement interval was available. Based on the last two measurements, the terminal basal area for each plot was linearly adjusted so the entire interval for all 12 plots was 35 years; an integer multiple of the simulated growth projection cycle. Differences between actual and simulated plot basal area was compared for the first 10 and last 25 years of elapsed time since 1942 separately for redwood and Douglas fir. Average initial plot conditions and the results from these comparisons are summarized in Table 15. Individual plot comparisons are tabulated in Appendix III.

[3/] These data were not used in model development.

Table 15. Comparisons of predicted versus actual basal area growth for 12 Union Lumber Company plots for redwood and Douglas fir. Summary statistics are expressed on a per acre basis.

Notation

A_i = Actual net basal area growth in square feet for plot "i"

P_i = Predicted net basal area growth for plot "i"

$D_i = A_i - P_i$

$R_i = A_i/D_i$

n = number of plots (12)

$$RMSD = [\frac{1}{n}\sum_i(D_i - \overline{D})^2]^{\frac{1}{2}}$$

$$RMSR = [\frac{1}{n}\sum_i(R_i - \overline{R})^2]^{\frac{1}{2}}$$

Average Initial Plot Conditions

Species	Basal Area (ft^2/Acre)	Age (Years)	Site Index (feet)	Trees (per Acre)
Redwood	100	29	110	117
Douglas fir	70	25	136	80

Redwood Comparisons

Time Interval	$\overline{A}$ (ft^2)	$\overline{D}$ (ft^2)	RMSD (ft^2)	$\overline{R}$ (%)	RMSR (%)	$\overline{D}/\overline{P}$ (%)	RMSD/$\overline{P}$ (%)
0-10 years	35.4	2.3	6.6	1.03	.20	.07	.20
10-35 years	72.2	3.9	25.1	.95	.30	.06	.36

Douglas fir Comparisons

Time Interval	$\overline{A}$ (ft^2)	$\overline{D}$ (ft^2)	RMSD (ft^2)	$\overline{R}$ (%)	RMSR (%)	$\overline{D}/\overline{P}$ (%)	RMSD/$\overline{P}$ (%)
0-10 years	25.3	-1.4	7.3	1.02	.25	-.05	.27
10-35 years	66.5	4.6	20.1	1.15	.38	.07	.33

In general, the model seems to be performing well in comparison to actual growth on these plots. The statistic $\overline{D}$ is the average difference between actual and simulated growth. Similarly, the statistic $\overline{R}$ is the average percent difference. If the model was "right on", we would expect these values to be 0.0 and 1.0, respectively. Considering that none of the heights or crown ratios were available at the first measurement, and consequently, had to be estimated to initiate the simulations, predictions are fairly well centered around actual growth after both ten and 35 years of simulation.

In mixed stands, species dynamics are an important management concern particularly when the species are as different as redwood and Douglas fir. An extensive examination of the growth and simulated development on several of these plots has indicated the model effectively mimics the interaction between these species. One plot that is good example of species interactions and growth differences for both actual and simulated development is shown in figure 19. The "actual" volume yields were estimated from total height-DBH volume equations with tree heights being estimated with a plot-based height-DBH-age regression equation. This stand was predominately redwood by basal area in 1942 (redwood basal area per acre, 79 ft^2; Douglas fir, 28 ft^2). Douglas fir however predominated in stems per acre (135 versus 50 for redwood). As is often typical of mixed-species stands in the region, the Douglas fir site index was 21 feet greater than redwood. This site index differential is presumably the major factor why the Douglas fir catches up with the redwood in basal area after 25 years and surpasses it there after. The relative differences in volume production are even more noticable. The greater volume in Douglas fir is due to its thinner bark and less taper

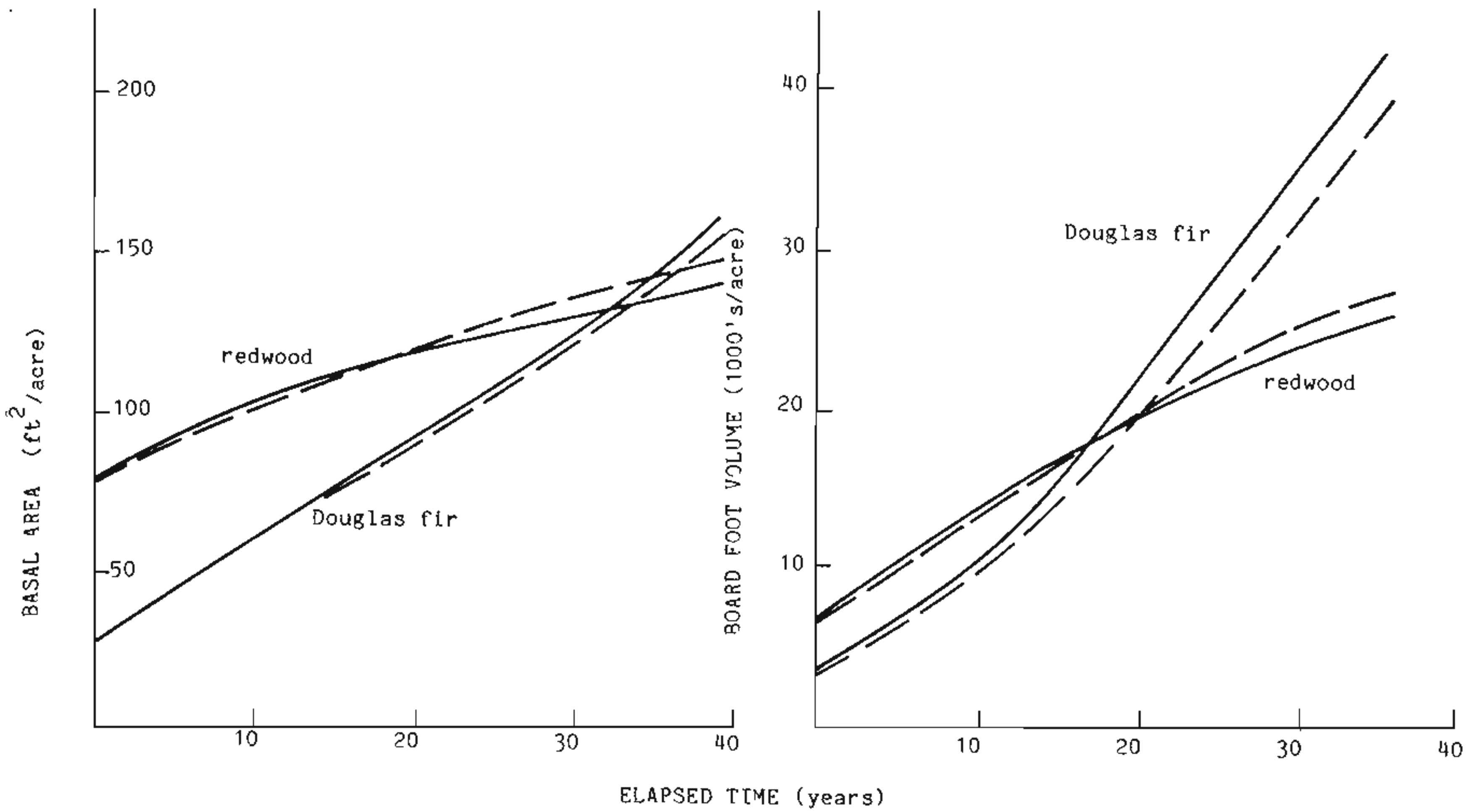

Figure 19. Actual (solid line) and simulated (dashed line) basal area and board foot volume yields for Union Lumber Company plot No. 7.

than redwood as well as a greater growth rate.

6.5.2 Jackson State Forest CFI Plots

In 1959, a continuous forst inventory (CFI) system of 144 one-half acre permanent plots was established on Jackson State Forest.[4/] These plots have been remeasured four times since on a five year cycle producing a twenty year measurement interval. Forty-three plots that were not harvested during this interval were used for comparitive purposes. Trees down to three inches DBH were recorded on these plots. Ingrowth trees recorded on remeasurement were ignored so that all comparisons were relative to the trees recorded on the initial measurement. At the initial measurement, approximately fifty percent of the trees had total height measurements enabling local height-DBH models to be constructed for each plot and species for subsequent use in estimating missing heights. All trees had crown ratios recorded. As opposed to the Union plot set, these plots had been systematically located an would tend to be the type of data resulting from an operational inventory.

Examination of the actual growth trends on these plots has revealed some anamolies that have not been given explicit consideration during the course of this analysis. Table 15 shows the average net and gross basal area growth and mortality for these plots for each five year interval. One noticeable factor, particularly for Douglas fir, is that mortality for the first five year period (which is right after plot establishment) is much less than the other periods. One hypothesis for this occurrence is that there may be a tendency for field crews not to

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4/ A complete description of the measurement procedures can be found in: Jackson State CFI System, 1959, State of Calif., Division of Forestry.

tag trees that seem to be "almost dead". Several field foresters substantiated this premise. As most of the data used in the mortality models were from plots during the first measurement interval after establishment, mortality may be underestimated.

Table 16. Periodic five year gross and net basal area increment and mortality for some Jackson State Forest CFI plots by species.

Redwood
(38 plots)

Calendar Period	Gross Growth	Net Growth	Mortality
	---- ft^2/acre ----		
1959-64	20.9	20.6	.3
1964-69	18.6	18.2	.4
1969-74	14.5	13.9	.6
1974-79	16.5	16.0	.5

Douglas fir
(20 plots)

Calendar Period	Gross Growth	Net Growth	Mortality
	---- ft^2/acre ----		
1959-64	12.1	11.8	0.3
1964-69	11.0	10.1	.9
1969-74	10.7	10.1	.6
1974-79	11.5	9.5	2.0

The most noticeable statistic in table 16 is the sharp reduction in both gross and net basal area growth for redwood in the third five year growth interval. Forest growth theory would suggest a slight gradual decrease in plot averages over time such as exhibited by the Douglas fir. Exhaustive checks however, have indicated no computational problems or the presence of any abnormal plots that are distorting the average estimate. Fifty-five percent of the redwood plots showed an increase in growth of more than 10 percent from the third to the fourth interval and only 15 percent decreased by more than 10 percent. Figure 20 shows actual net and gross average plot growth by period for each species and simulated growth based on projections made from trees at plot establishment. Comparative statistics are summarized in table 17 and individual plot comparisons are tabulated in Appendix III. Also shown in figure 20 is the average simulated plot growth based on the actual mortality records of each plot[5/]. There are minor differences between these two modes of simulation. However, the latter helps explain the underpredictions for Douglas fir in the first interval and the overpredictions in the fourth interval.

5/ In these simulations, all of the four tree records associated with a given initial tree were deleted from the internal tree list if the tree had been recorded as dead.

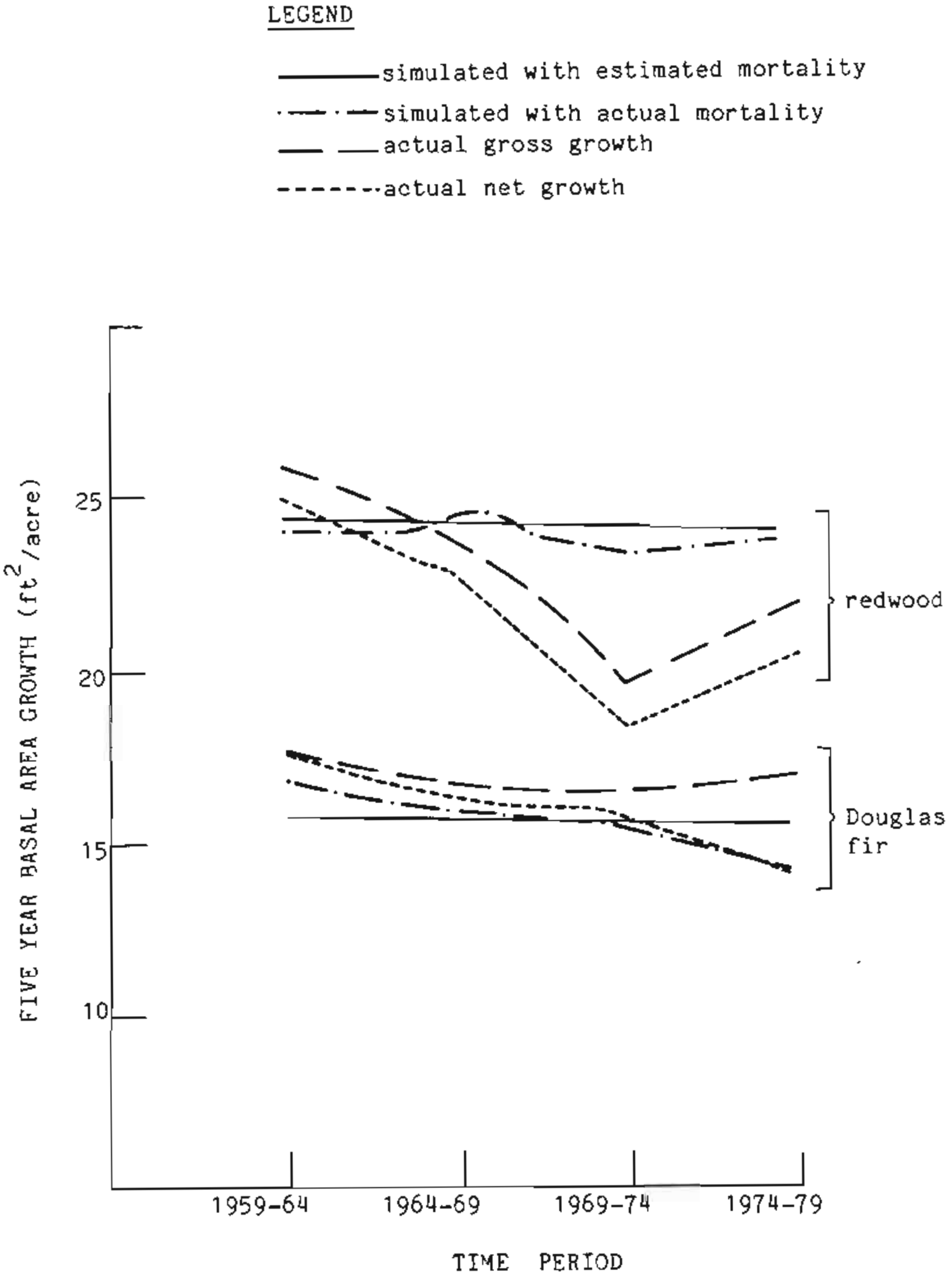

Figure 20. Average actual and simulated basal area growth for selected growth plots from Jackson State forest

Table 17. Comparisons of actual versus predicted basal area growth for 43 1/2 acre Jackson State Forest CFE plots for redwood and Douglas fir. Statistics are expressed on a per acre basis. Notation is the same as Table 15.

Average Initial Plot Conditions

Species	Basal Area (ft^2/Acre)	Age (Years)	Site Index (feet)	Trees (per Acre)	Number of plots
Redwood	178	43	114	194	38
Douglas fir	64	37	146	61	20

Redwood Comparisons

Time Interval	$\overline{A}$ (ft^2)	$\overline{D}$ (ft^2)	RMSD (ft^2)	$\overline{R}$ (%)	RMSR (%)	$\overline{D}/\overline{P}$ (%)	RMSD/$\overline{P}$ (%)
0-5 years	20.6	.4	5.7	1.06	.28	.02	.28
5-10 years	18.2	-1.8	4.7	.93	.26	-.09	.23
10-15 years	13.9	-5.9	6.0	.75	.25	-.30	.31
15-20 years	16.0	-3.4	5.6	.87	.31	-.17	.28

Douglas fir Comparisons

Time Interval	$\overline{A}$ (ft^2)	$\overline{D}$ (ft^2)	RMSD (ft^2)	$\overline{R}$ (%)	RMSR (%)	$\overline{D}/\overline{P}$ (%)	RMSD/$\overline{P}$ (%)
0-5 years	11.8	2.5	2.6	1.37	.45	.26	.28
5-10 years	10.1	.54	3.5	1.18	.50	.05	.36
10-15 years	10.1	.4	3.0	1.11	.49	.04	.31
15-20 years	9.5	-.12	3.3	1.15	.47	-.01	.34

Redwood growth behavior, however, is still puzzling and it would seem that some widespread unknown factor(s) associated with calendar periods is influencing redwood tree growth to a much greater extent than Douglas fir. As an alternative check, a transect was made across Jackson State Forest during 1978 and six sampling locations were established. At each location, four codominant redwoods and four codominant Douglas fir trees were selected that appeared to have historically developed in uniform environments. Increment cores were extracted and radial increment measured for each tree from 1950 to 1977. Average radial increment of each species is graphed in figure 21 against calendar year. Growth theory would suggest that radial increment would gradually decrease over time for these trees. However, there is an apparent peak in growth of both species during the early 1950's. Lindquist and Palley's data came from this period and may be one reason why their growth estimates are much higher than the model developed in this study. The patterns for both species from 1959 to 1977 conforms to the actual plot growth shown for the Jackson State CFI data. Attempts to correlate these patterns with rainfall or cone crop ratings [6/] has not pointed to these sources as possible explanatory factors.

Cyclical patterns in tree growth have been noted for tree species in other regions (Lephart and Stage,1971, Jonsson and Matern, 1978). If periodic cyclical fluctuations in tree growth of the magnitude shown here are typical of the historical growth patterns of redwood, it would cast some doubts on the precision of any form of growth model for the purpose of short term growth prediction and may limit the effectiveness of any calibration scheme based on past short term growth measurements. This latter aspect is discussed in more detail in chapter 7.

In summary, the model performs reasonably for the Douglas fir component in this comparison. Redwood comparisons however indicate that while the general level of the predictions seem to be within range of what is actually occurring, unexplained short term differences in

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6/ In years of high cone production, much of the trees photosynthetic production is diverted to cone growth rather than asexual parts.

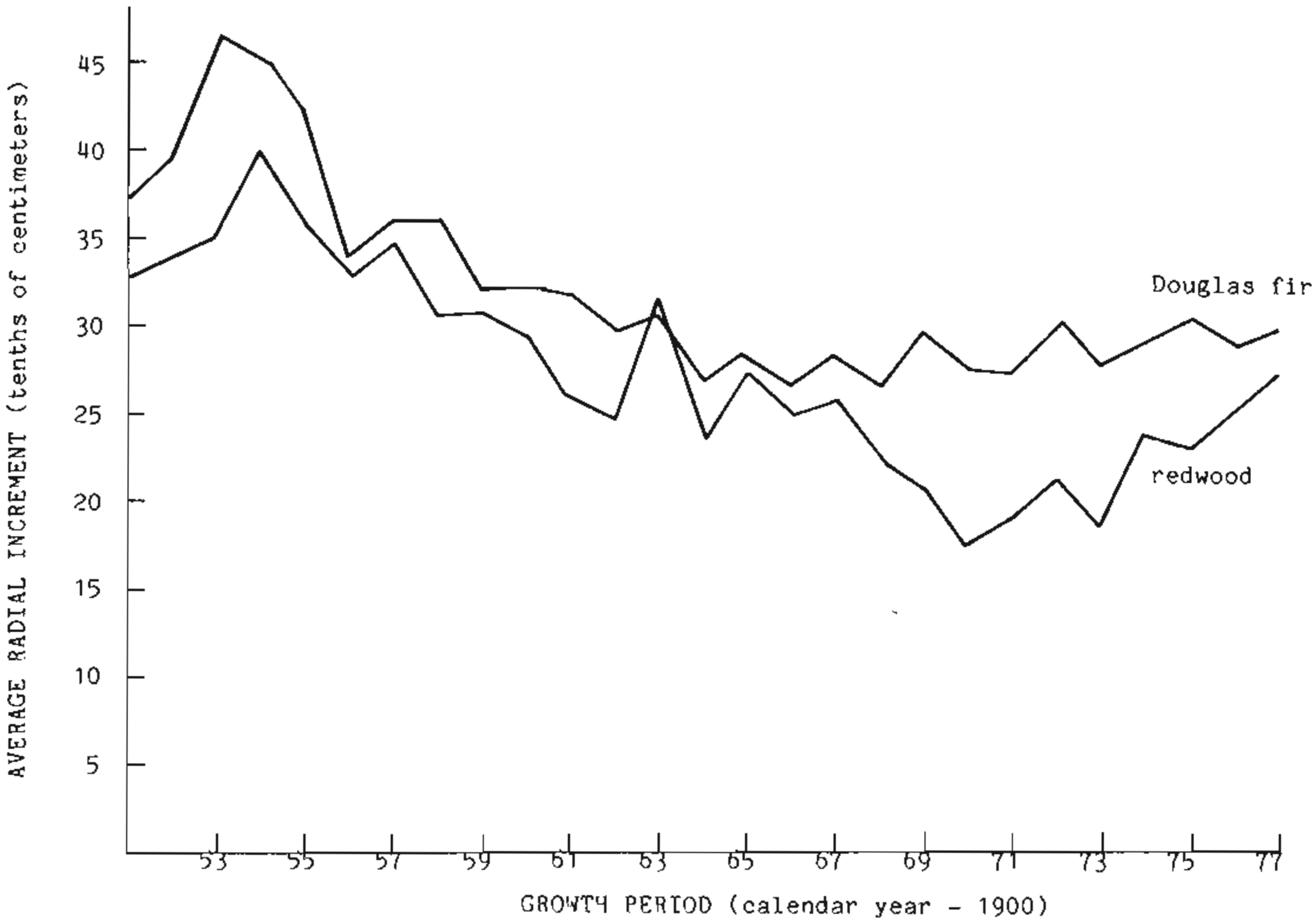


Figure 21. Average annual radial increment for 20 redwood and 20 Douglas fir trees collected at five locations on Jackson State Forest.

predicted versus actual growth trajectories do exist.

6.5.3 Arcata City Park Plots and the Wonder Plot

In 1922, four one half acre growth plots were established in the Arcata City Park. Two of the plots had been moderately thinned from below prior to establishment. Subsequently, the plots were measured on a 10 year cycle. Forty years of growth records were available for comparative purposes. One plot was not used because of excessive blow-down recorded on the 1932 and 1942 remeasurements. Another plot had substantial blowdown in the last twenty years of the comparative record and was used as an indication of how sometime erratic patterns of tree mortality can distort predictions. In 1923, a one acre plot was established in an almost pure redwood stand on an alluvial flat of the Big River in Mendocino County. This plot has been named the "Wonder Plot" by Fritz (1945) because of its unusually high stocking. It was 64 years old in 1923 and had 598 square feet of basal area.

All of these plots had subsamples of total height measurements taken at establishment and were used to construct local height-DBH models to estimate all tree heights for simulation initialization. Crown ratios were estimated from models in the stand generation program. Growth was simulated on these plots for forty years and basal area yields are shown in figure 22 along with actual yields. The model performs quite well for these plots over this time period with most of the departures from predicted values being due to mortality.

6.6 Evaluation of Simulated Stand Response to Thinning

Being able to predict stand response to thinning was a major objective in developing this model system. However, there is a paucity of adequate response data for stands subjected to different thinning

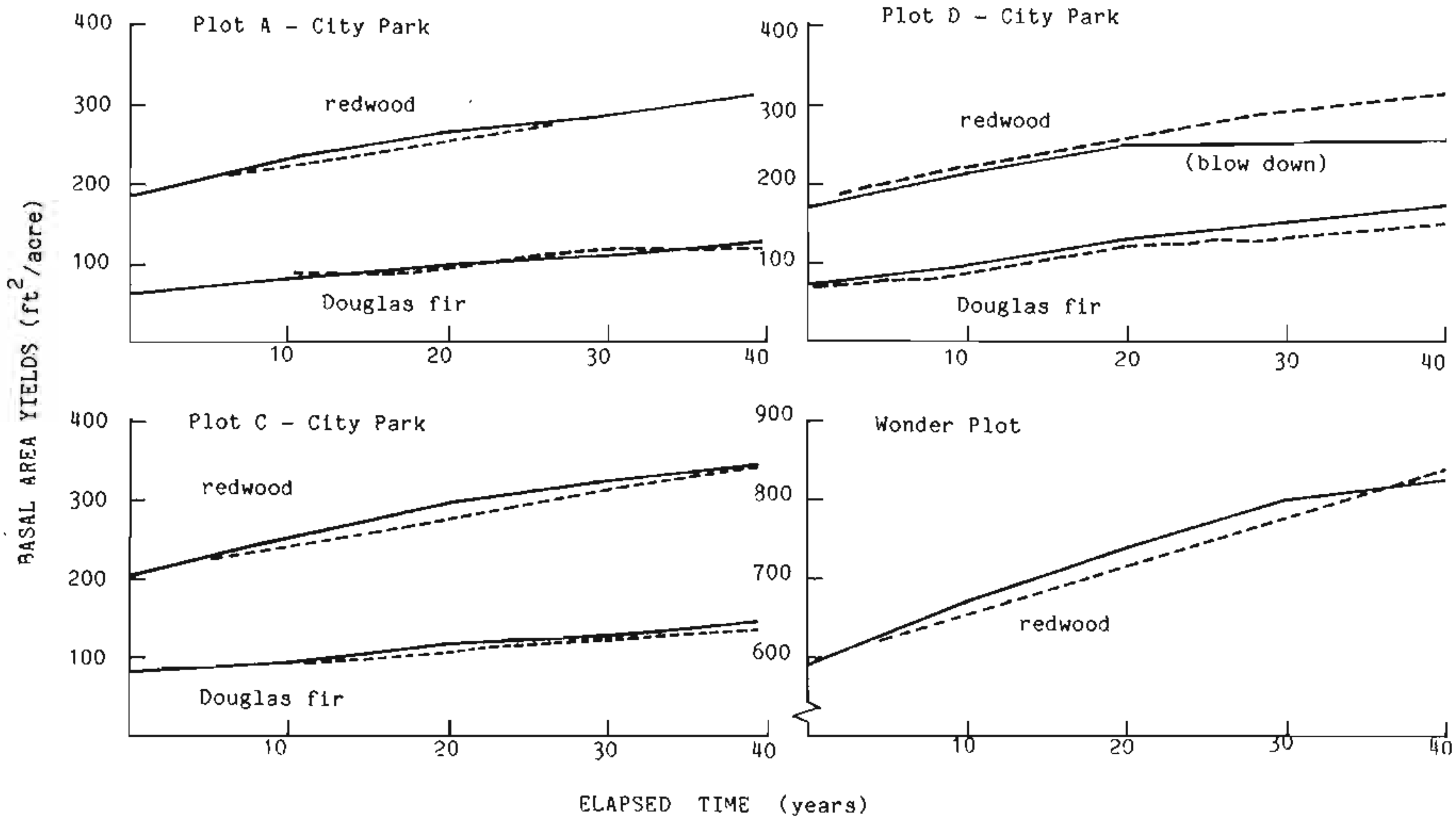


Figure 22. Actual (solid line) and simulated (dashed line) basal area yields for Arcata City Park plots and the Wonder Plot.

methods for which any definitive comparisons can be made. Several thinning experiments have been installed in the region to provide this kind of response information but sufficient time has not yet elapsed to produce usable growth records. Consequently, it will be some time before simulations of stand response to various thinning prescriptions can be evaluated. There are, however, some general tenets regarding thinnings that can provide some perspective to judge model behavior under different forms of partial harvests.

The practice of thinning has evolved to include several objectives. These include: removal of trees likely to die or of poor form to favor the trees in the residual stand; control of species composition for biological or economic reasons; salvage of dead or diseased trees; and numerous others. Some of these objectives include aspects of timber growing that are outside the scope of this study as they involve factors that are not modelled. The aspect of thinning that is relevant to the evaluation of the model system concerns the nature and degree that thinnings effect growth-density relationships. While several broad generalizations have emerged in this area in the literature, most of them are derived from an hypothesis introduced by Langsaeter (1941). He used cubic feet per acre as a measure of density and growth in cubic feet per acre per year to illustrate five types of growth-density relationships:

Type I Growth is an increasing linear function of density. Trees are so far apart that there is no inter-tree competition.

Type II Inter-tree competition begins which results in a declining rate of increase in growth with respect to density.

Type III Growth reaches a maximum and is virtually constant over a broad range in density.

Type IV-V Competition becomes severe and growth begins to decline with further increases in density.

These types of growth-density relationships are shown in figure 23. The kind of growth-density relationships depicted in the upper end of type II and type III zones form the basis of a generally accepted tenet of thinning: Various forms of improvement and salvage operations can be carried out in a stand with little appreciable decrease in growth so long as the density of the stand is maintained in a type III or high type II situation (Smith,1962). Implicit in this tenet is the assumption that the trees left after a thinning are the more vigorous trees. Removal of the larger and more vigorous trees in the stand is considered a poor management practice because the trees that are left usually have small crowns and are of such poor vigor that their potential to respond to the increase in growing space after thinnings is limited.

These general tenets of thinning provide some general expectations of how stands should respond to thinning treatments. The model system has been used to examine several levels of thinning intensity in redwood and Douglas fir stands of different densities and sites at different ages. Two methods of simulating thinnings have been used:

Method B This is a low thinning (from below) where, for a specified amount of basal area to be removed, all the trees in a two inch DBH class, starting from the smallest trees, are removed before thinning in the next two inch size class.

Method A This is a high thinning (from above) method that does the opposite of method B; trees are removed starting with the largest.

The effect of both thinning methods is to truncate the stand DBH

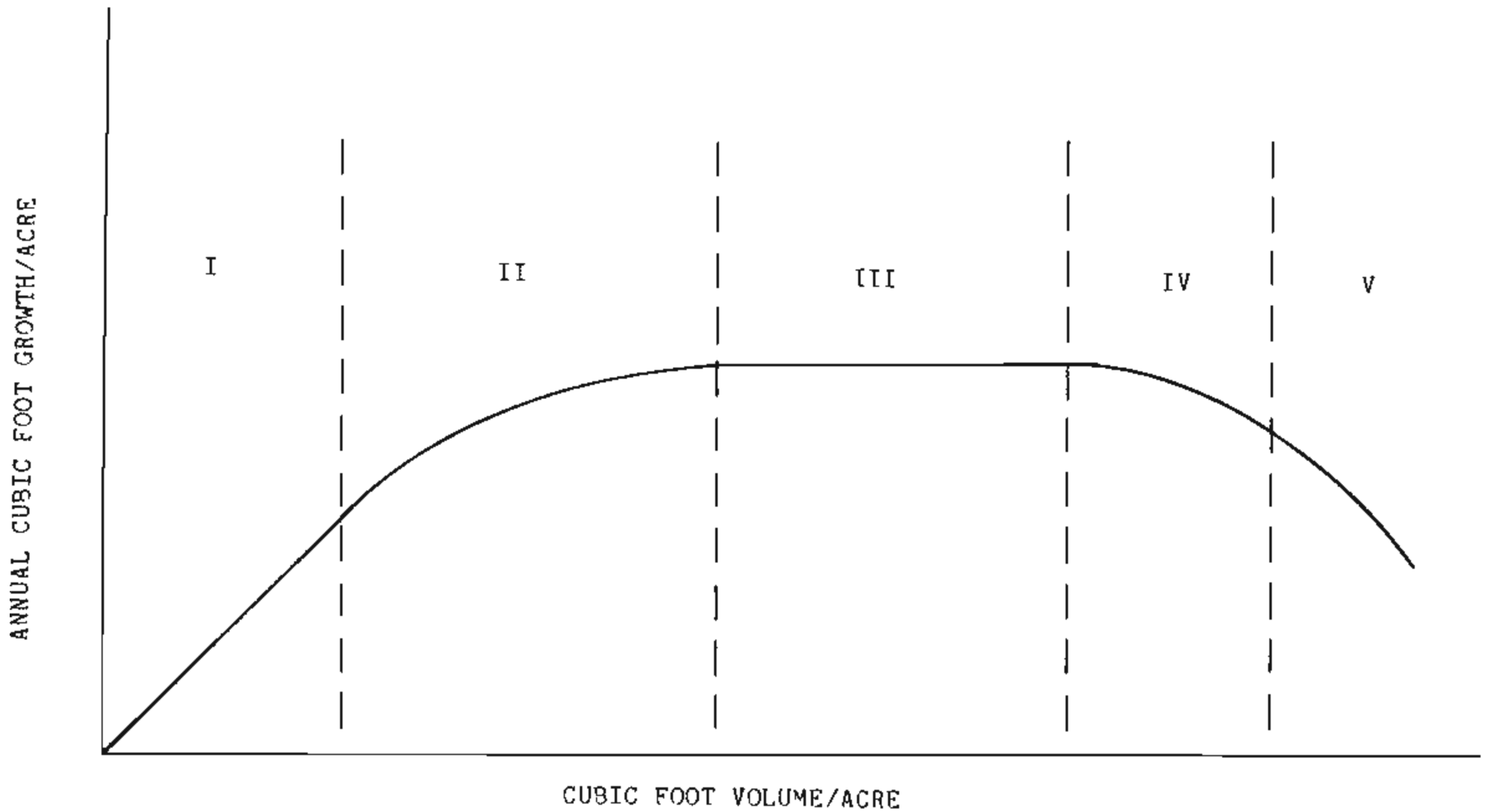
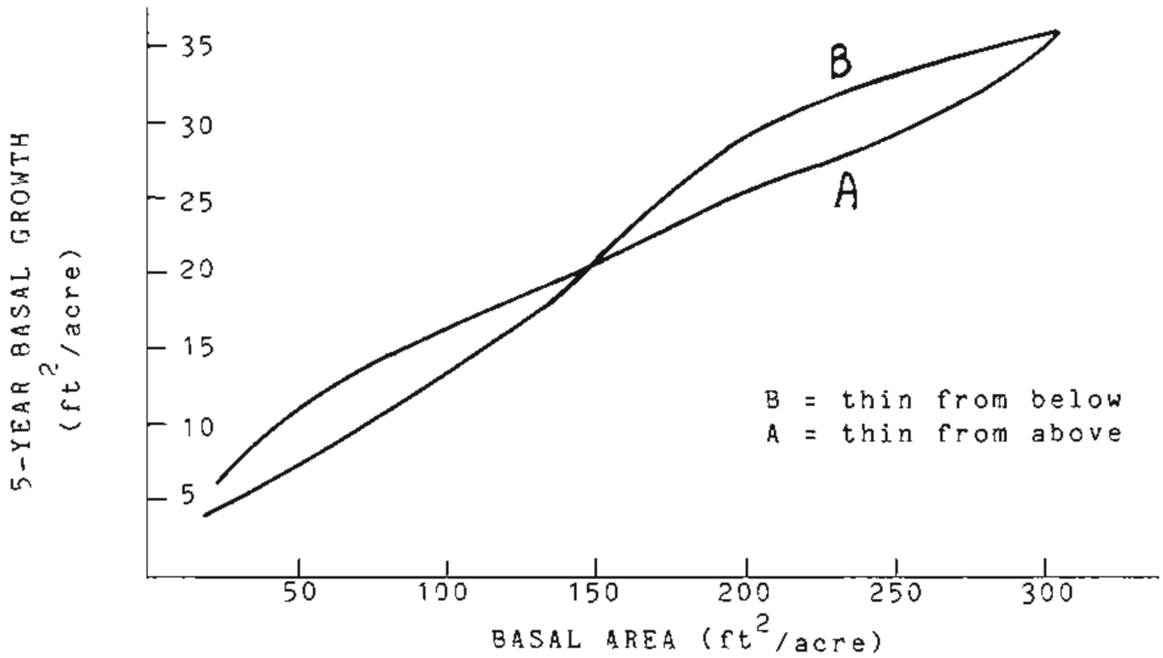


Figure 23. The relationship between growth and stand density. (after Langsaeter, 1941)

distribution from either tail. Strict adherence to size characteristics is not the usual way stands are thinned as factors such as tree form, spacing, thriftiness, and merchantability enter into field marking decisions. It was used here mainly to examine growth-density relationships.

In none of the several thinning simulation experiments that were performed did any growth-stocking relationships appear that were counter to the tenets previously described. Simulated response of Douglas fir stands thinned at different intensities by method B were an almost classical example of the growth-stocking relationships hypothesized by Langsaeter. Growth-stocking types I-III were generally present in thinning response functions for stands that were moderate to well stocked prior to thinning. In some very dense conditions, the modelled response indicated slight increases in net growth after thinning compared to the unthinned state. Simulations for redwood however are quite different. Growth-stocking simulations of redwood in either a thinned or unthinned state indicate that stands never get beyond a zone of type II. Simulations for stands at the highest density levels found in sample plots in the data base show that maximum growth rates have still not been attained. Lindquist and Palley (1967) have also indicated that at even the most advanced age for young-growth timber and at the highest level of stand basal area likely to be found in coastal redwood stands, both basal area and volume periodic growth have not yet reached a maximum with respect to density.

The top diagram in figure 24 shows basal area growth response curves for a site 110 redwood stand that was generated at age 10 with 400 stems per acre, grown to age 35, and thinned to different residual stocking levels by methods A and B. Method B indicates an increasing



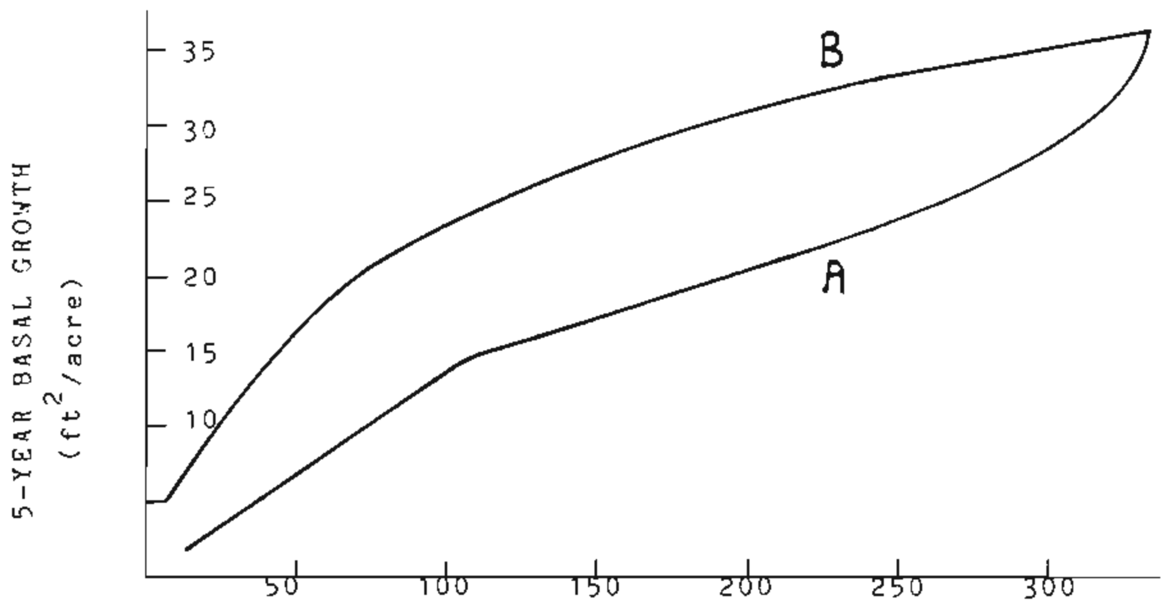


Figure 24. Simulated five year growth in basal area for a 35 year old redwood stand subjected to different thinning methods and intensities

rate of decrease in growth with decreases in density expressed as basal area and is similar to type I-II growth-density zones shown in figure 23. In conformance with the tenets, there is greater growth for method B than method A until residual basal area drops to about 150 square feet per acre. At this level, the residual stand thinned under method B is understocked and not fully occupying the site. While the residual stocking in terms of basal area is the same for stands thinned under each method, there are fewer stems per acre for the stand thinned under method B than method A. The second diagram in figure 24 shows growth after thinning as a function of residual stems per acre and clearly shows greater growth for thinning method B if density is expressed as stems per acre. The same simulation was performed for a pure Douglas fir stand on a site index of 125. Results are shown in figure 25. The results are similar to those for redwood but the decline in growth for moderate levels of thinning is not as pronounced as redwood.

These stands are relatively young in the chronology of a forest and stand crown class differentiation, which tends to enhance the growth of dominant trees and hampers the growth of intermediate and suppressed trees, has not been fully realized. Effects of differentiation become more pronounced in older stands and result in a further divergence in growth-density response functions for the two different thinning methods. The original ten year old stands were grown to age 80 and subjected to the same harvest simulations previously described. Harvest response curves showing five year growth in basal area and cubic foot volume as a function of density expressed as residual stand basal area are shown in figures 26 and 27 for redwood and Douglas fir respectively. Results are similar to the 35 year old stands but differences between

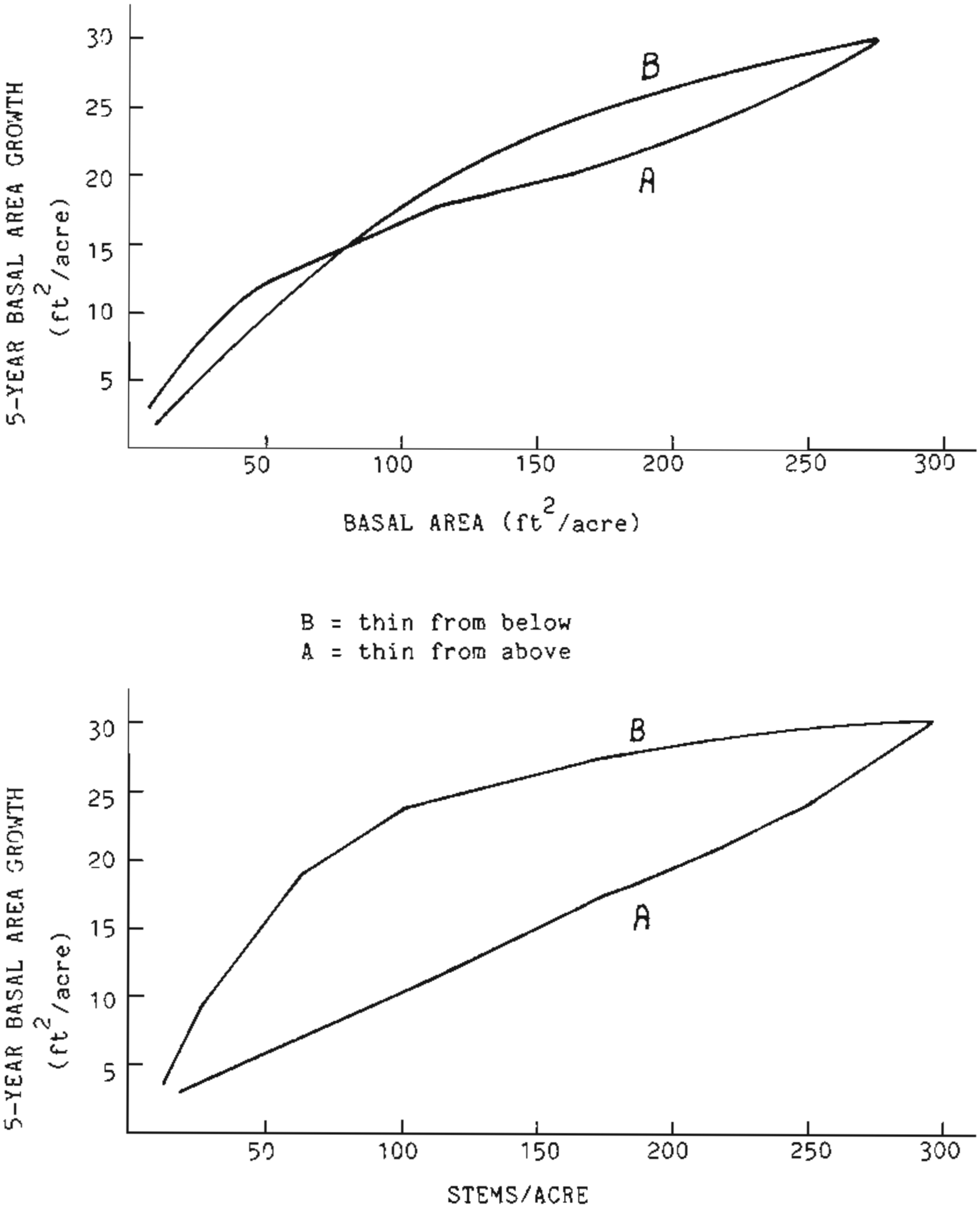
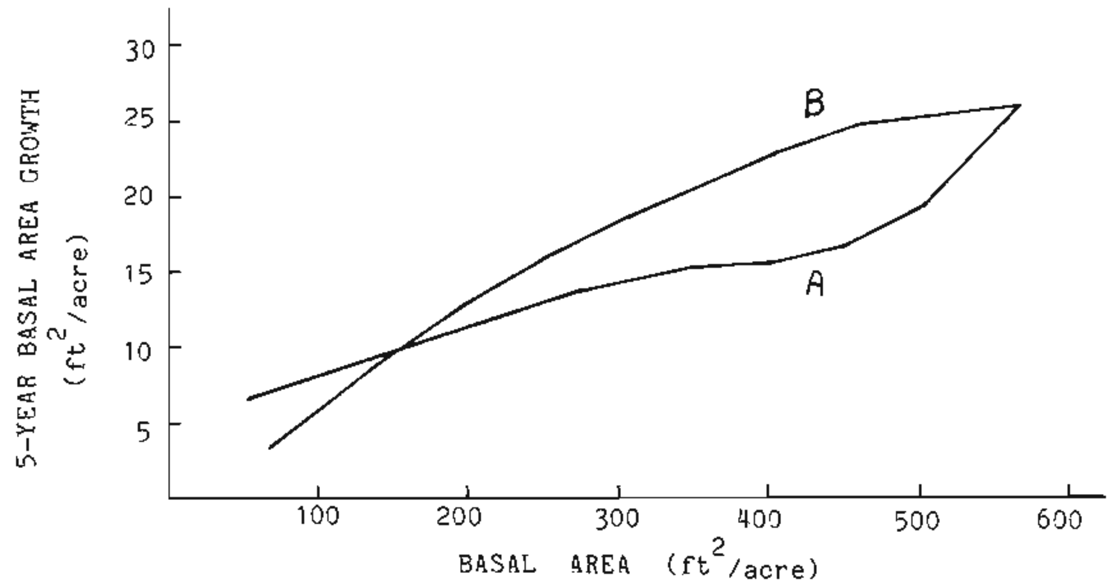


Figure 25. Simulated five year growth in basal area for a 35 year old Douglas fir stand subjected to different thinning intensities and methods.



B = thin from below
A = thin from above

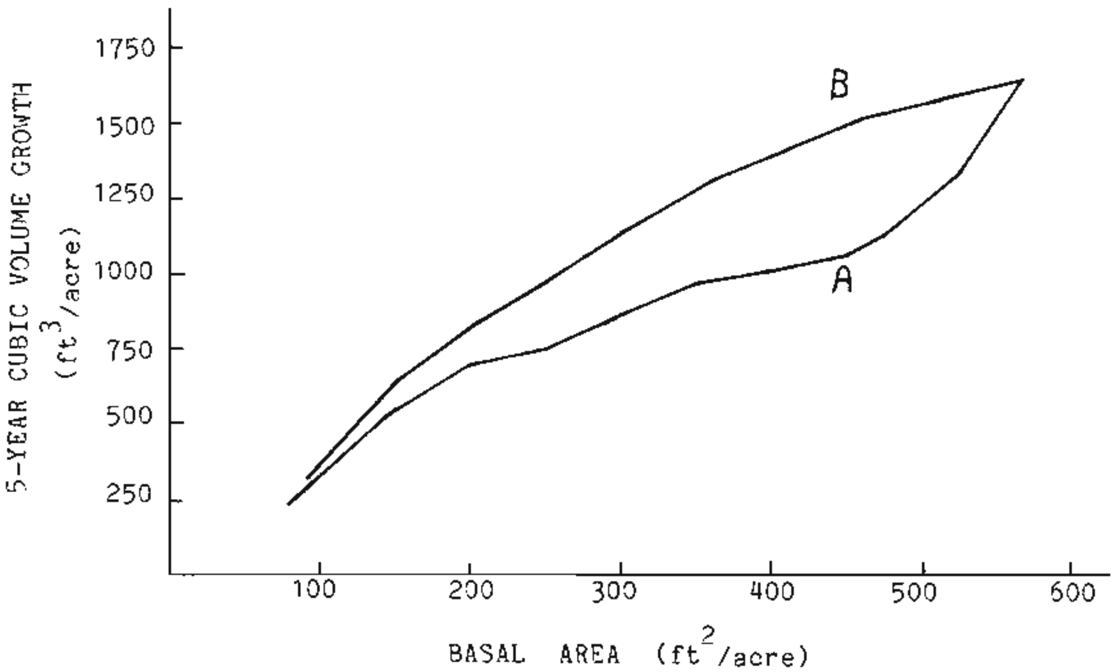
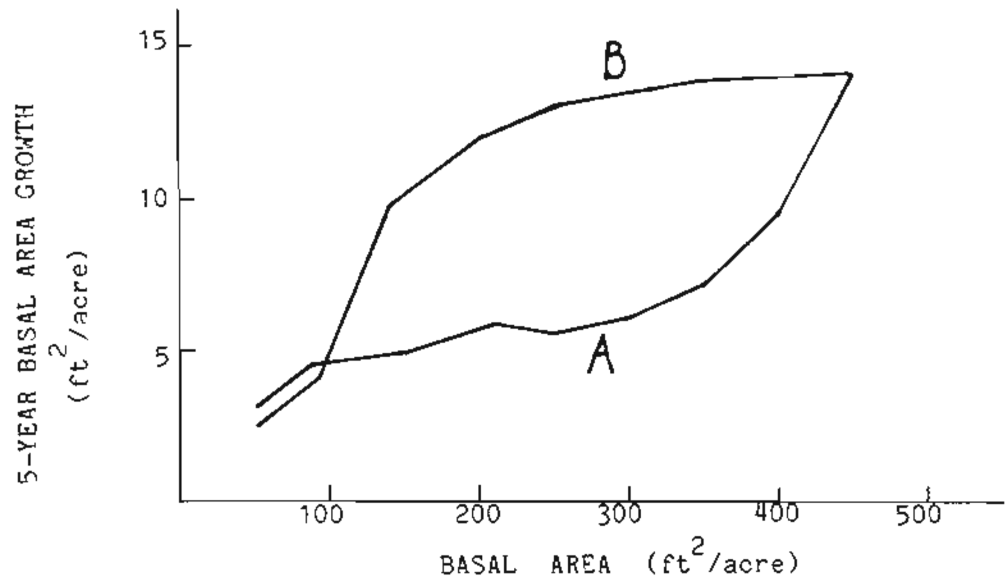


Figure 26. Simulated five year growth in basal area and cubic volume for an 80 year old redwood stand subjected to different thinning methods and intensities.



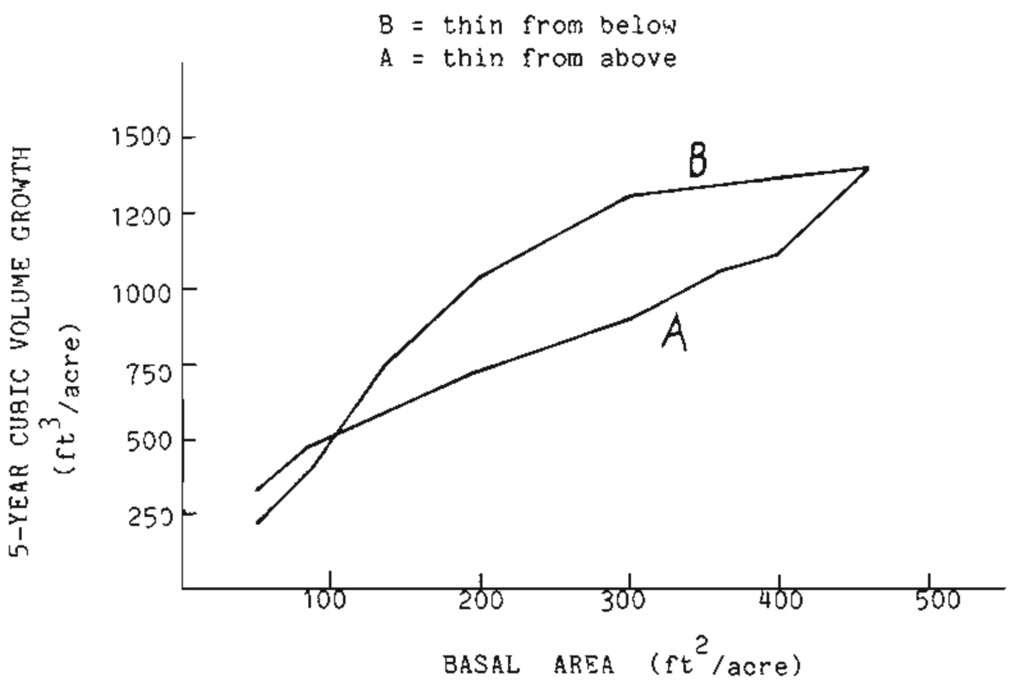


Figure 27. Simulated five year growth in basal area and cubic foot volume for an 80 year old Douglas fir stand subjected to different thinning methods and intensities

thinning methods are more accentuated. In particular, the Douglas fir simulations indicate an almost constant growth rate for a wide range in moderate thinning intensities for thinning method B and would be indicative of a type III response zone.

Differences in the shapes of the thinning response functions of the two species are speculated to be due to differences in shade tolerance. Redwood, being more tolerant than Douglas fir, has a greater capacity to grow in dense stand conditions and is presummably a reason why increases in stand density are not totally offset by a decrease in net stand growth. The almost negligable predicted mortality rates for redwoods in old stands also contributes to increasing net growth with increasing density.

An alternative thinning method was also tested for these and other simulated stand conditions that removed increasing amounts of basal area based on the assigned diameter growth equation modifier rather than size characteristics. Removing trees with the smallest equation modifier first produced response functions similar in shape as method B but slightly greater for the same residual basal area. Conversely, removing trees with the largest equation modifiers first produced response curves similar in shape but slightly lower than method A. Moreover, the shapes of the residual stand DBH distributions tended to be more in line with what would occur with the classical definition of a high or low thining; the distribution has the appearance of being "shaved" at either tail rather than truncated. This method was motivated out of a consideration of what actually goes on during field marking. In a low thinning, the smaller and less vigorous trees are marked for cutting to favor the larger and more thrifty trees. While size can be used as a harvest simulation criterion, tree appearence or field evaluations of tree vigor cannot be directly used. Hence, the equation modifier was substituted as a proxy for a field judgment of tree vigor. Simulations have indicated that this method may add a greater sense of realism to thinning simulations. However, some additional experimentation and analysis is necessary to determine if field judgments of poorer growing trees are generally accurate before routine use of the method can be made. It is an interesting area for future study.

In summary, the model seems to give reasonable predictions of stand response to thinnings. It is emphasized that this aspect of model operation is an emergent property of the implemented tree growth system and is yet unsubstantiated by empirical response data. Hence, simulated thinning responses should be considered interim measures until they can be tested against experimental thinning data.

6.7 Evaluation Summary

The model system has been implemented and the individual components have been examined separately without any signs of unreasonable behavior. Simulations with the system produced growth and yield estimates that were in conformance with general tenets of forest growth. Similarly, comparisons of model simulations with recorded plot growth indicated the general level of the predictions were in conformance with actual development data.

Chapter 7

LOCAL CALIBRATION CONSIDERATIONS

The model system as implemented provides reasonable projections of growth and yield throughout the range of conditions likely to be encountered in young growth stands in the region. One final aspect that will be dealt with briefly is a consideration of using historical growth data to calibrate the model system to local conditions.

Theory and methods of locally calibrating growth projection systems is a subject area that has received little attention in the literature. While this topic is of sufficient breadth and importance to deserve special study in its own right, the treatment presented here is intended to outline some of the aspects of calibration that should be considered.

7.1 The Calibration Environment

A situation in which the model system may need to be adjusted may arise from having some stand performance data that differs in some respect from the predictions obtained from the model. These types of data may come from CFI systems that are used to moniter property-wide changes in forest conditions or from increment corings taken in conjunction with stand inventories. The type of growth information that is likely to be available for this purpose is very limited and it will probably limit calibration to basal area growth. However, to some extent, the height growth functions can already be considered "calibrated" because the predominant explanatory variable is site index which is based on past growth.

7.2 Forecasting Objectives in Relation to Calibration Decisions

The decision to calibrate and factors which may influence the calibration method are to some extent contingent on the objectives of forecasting. If the objective is to compare alternative treatments for the same stand, and the premise is accepted that the model is correct up to a multiplicative constant, then the relative ranking of any management scenarios will not be affected much by a change in model scale. In these types of situations, calibrating the model may be unnecessary because it will not change the rankings.

If the projection is to update past inventories to the present, then the concern is in an absolute estimate of growth which may be improved by locally calibrating the model. In updating past inventories with concurrent past growth data as calibration material, periodic fluctuations in tree growth are not a concern as they are presumably reflected in the past growth data and it is desirable to incorporate their effects in estimating past absolute growth.

When considering to calibrate the model system to more accurately reflect the probable future course of development for a given stand, possible periodic fluctuations in tree growth may need to be considered. While individual future periodic fluctuations cannot be predicted, a reasonable projection objective would be to calibrate the model so that the future projections are an extension of past long term trends. Consequently, the calibration data set from a short period of time needs to be assessed to see whether it is also representative of long term trends.

7.3 A General Proposal

In developing a calibration strategy, there are numerous possibilities for modifying the model system to account for departures of predictions from actual growth. For example, all or some of the plot and tree calibration functions used to develop equation modifiers could be reestimated from a local sample or the entire model system could be reestimated. As a routine practice, however, these possibilities may not be cost effective considering the problems one would encounter and the necessary data requirements (cf., chapter 5).

As a routine procedure that is amenable to machine processing and repetitive applications, it is proposed that the model system itself be unaltered. Rather, the predictions themselves would be altered based upon a local sample of basal area growth. For this, a simple ratio might be used. That is, if the predictions are say 10% high for the calibration plots and we are willingly to assume the model is correct up to a multiplicative constant, the predictions for all plots could be reduced by 10%. More formally, we let[1/]

$$y_i = Rx_i + e_i \tag{7-1}$$

where

y_i = actual basal area growth for the i^{th} plot

x_i = predicted basal area growth for the i^{th} plot

R = multiplicative adjustment to be estimated

e_i = random error term

Equation 7-1 can be used as a basis for several possible sampling designs which may range from specific stand to entire properties in scope. There are several possible estimators for R. The efficiency of any of these estimators will depend on the specific calibration

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1/ This proposal is assumed to be species specific.

criterion and assumptions regarding the distributions of the error terms. While the simplest estimate of the ratio R is the ratio of means,

$$\hat{R} = \frac{\text{average actual basal area growth}}{\text{average predicted basal area growth}} \qquad (7\text{-}2)$$

Raj (1968) discusses the properties of alternative ratio estimators.

It is suggested that the component for estimating R be based on gross rather than net growth as mortality is often erratic and may reduce the precision of any estimates.

When it is desired to calibrate the model to forecast future growth, it is proposed that adjustments be made relative to the past long term growth norm for the area. This judgement presumes that cyclical fluctuations in tree growth will be distributed about this norm in the future. As a means for determining a past local long term growth norm, development of a ring-width index such as was done by Jonsson and Matern (1978) could produce a historical trend in tree growth by calendar year. Such an index could be used to correct for short term fluctuations in tree growth in the calibration data set. The general proposal previously described could then be implemented.

As a final note, a major decision that has to be addressed before any adjustments can be made is what form of evidence is sufficient to indicate that the model is in need of adjustment? A classical statistical approach would involve a test of the hypothesis that calibration factor "R" is equal to 1. If the hypothesis is accepted, the model would be used without adjustment. If the hypothesis is rejected, the model would be adjusted by the sample based estimate $\hat{R}$. Quite obviously, acceptance or rejection would involve some form of decision criterion. While a Bayesian approach might be considered, it is emphasized that there are numerous factors involved in a calibration decision that are not totally placed in the framework of an objective analysis. Has enough care been exercised in the collection of calibration samples to justify a calibration analysis? Are the samples representative of the population to which the calibrated model is to be applied? Ultimately, users of the model have to be thoroughly convinced that their calibration material is adequate before calibration decisions can be made.

In summary, a general and simplistic method has been proposed to calibrate the model system to local conditions. While being untested, it seems reasonable based on impressions gained during the course of this study and should be an adequate provisional measure until the subject can be studied in greater detail.

Chapter 8

SUMMARY, CONCLUSIONS, AND EXTENSIONS

This study has focused on the development of a forest growth and yield projection system that can be applied to young-growth forest stands in the North Coast region of Caifornia. Research emphasis has been directed at two major but related areas. The first has been the design and implementation of a model system that is capable of satisfying a set of pragmatic user-oriented operational objectives. The second has been the consideration of certain theoretical and methodological aspects that ultimately effect the consistency and overall validity of the model system.

8.1 Operational Objectives

The four main operational objectives described in Chapter 2 were considered to be accomplished in this study.

1) Designing the projection system as a species-specific, tree-based, distance-independent growth model allows a direct application to specific timber stands as the measurements used in quantifying timber stands are used as direct model inputs.
2) As a link in a forest planning system, the system is compatible with inventory systems as the basic measurements collected in the inventory process are also used as primary model inputs. Similarly, as predictions of tree-by-tree inventory measurements are the basic output variables of the projection system, this

information can be aggregated into a form consistent with decision-making requirements.

3) The system is also capable of simulating a wide variety of partial harvests as the internal tree list is directly accessible in model operation.

4) The model system is also capable of being calibrated to specific stands of trees and a tentative procedure for accomplishing this has been proposed.

As a practical substantiation of this effort, the computer program in which the basic model system is imbedded has been released to several foresters actively engaged in young growth forest management and the general consensus is that model performance is quite realistic.

8.2 Theoretical and Methodological Contributions

A major theoretical contribution of this research has been the characterization of tree growth processes as a conceptual model system with three main attributes: 1) it is a simultaneous equation system, 2) it operates in a recursive manner, and 3) it combines both time series and cross-sectional phenomena. This latter aspect led to the adoption of a random coefficient regression model as a means of accounting for both types of variation in tree growth.

Interpretive differences in the model system resulting from potentially analyzing an identical data set in different ways were subsequently discussed. The type of analysis leading to what was termed a type B model was identified and interpreted as providing a description of the current stand condition. This was contrasted with an analysis that would produce a type A model or one that can be interpreted as describing the process that generated the stand. It was concluded that a model system with type A attributes was most consistent with observed phenomena of tree growth and the of properties that are considered desirable features of the growth projection system adopted for this study. Development of type A model system was subsequently adopted as an analytical goal.

Problems in attaining this goal were largely attributable to data inadequacies and necessitated the use of two main ad hoc techniques. For height growth model development, an extraneously-derived growth trajectory was used as a primary explanatory variable. For the diameter growth model, a data stratification method was employed to reduce potential biases in model coefficient estimates. It was shown that this procedure produced different growth trajectories than a type B analysis with the difference being in a direction that was theoretically anticipated and consistent with a type A model. Direct tests to determine whether this ad hoc procedure produced results that were statistically equivalent to results obtainable from a theoretical type A analysis were not made due to lack of data. Indirect evidence, however, in the form of comparisons of model projections with actual plot growth indicated that the model performed reasonably.

The development of the canopy cover vector was also a major methodological contribution of this study. This device was shown to provide a logical and biologically interpretable density index for use in developing competition factors in distance independent tree growth models.

It is concluded that the operational and methodological objectives of this study have been satisfied. It is emphasized however that the model system developed in this study is a preliminary effort with numerous possibilities for refinement and extensions.

8.3 Extensions and Refinements

Many of the possible extensions and refinements of the growth projection process are contingent upon the acquisition of a better and extended data base to be used in analysis. The organizational means under which this data could be acquired is a subject matter worth investigating in its own right. However, data availability is currently the most limiting factor in further model refinements. Of several possible avenues for additional research, the following items are considered to be of major importance.

8.3.1 Canopy Vector

Use of crown cover precent as a competition measure assumes that crown radii are solely a function of the distance from the tree tip and the relationship was the same for all species. Moreover, given tree height and crown ratio, all trees irrespective of species were assumed to offer the same amount of competition to other trees. These assumptions were not fully verified and further investigations in this area may lead to a greater resolution of species interactions in mixed stand dynamics.

8.3.2 Estimation Method Refinements

A premise of Chapter 3 was that time series information of sufficient length to allow the component models to be fitted to individual trees was necessary for a theoretically defensible implementation of the model system. As these types of data were unavailable, some *ad hoc* procedures were employed instead. If time series data ever become available, comparisons of the ad hoc methods used in this study with an ideal procedure would be invaluable, not only in verifying the adequacy of the model system as implemented in this study but also in terms of providing recommendations to other modellers working with insufficient measurements and response data.

8.3.3 Harvesting Schemes

Partial harvests are accomplished in model operation by removals of entire tree records or a reduction in per acre weights. Any harvest simulation scheme that relies on tree characteristics (species, height, DBH, crown ratio) maintained in the model would seem to be pragmatically defensible. However, one general objective of thinnings in intensive young growth management is to remove trees of poor form and vigor to favor a leave stand of healthy crop trees. To the extent that this type information used in field marking decisions is based on non-modelled tree characteristics, using internal equation modifiers as proxies for these characteristics may be a promising avenue to pursue in developing harvesting schemes.

8.3.4 Crown Recession Models

As crown lengths and crown ratios are primary explanatory variables in predicting individual tree growth and in aggregate, are used to develop competition measures, proper functioning of the crown recession models are crucial to the reliability of the model system. The crown recession data available for this study is at best marginal and as indicated in Chapter 6, extrapolation of growth projections beyond the practical upper limit of stand ages used in developing the models indicates that recession rates may be excessive. While the model was not intended to operate at these advanced ages, this observation may be indicative of some inadequacy in the structural form of the model. Development of an acceptable theory of crown recession at a resolution level compatible with the level at which the model system operates as well as incorporating a refined and extended data base is considered to be another avenue of future research.

8.3.5 Mortality Models

As indicated in previous chapters, the mortality models used in this study may underestimate mortality rates. Most of the observations on tree mortality were taken from plots the first measurement period after establishment. As conjectured in conjunction with the comparisons of model performance with measured growth on the Jackson State Forest CFI plots, mortality measured during this period may not be representative of long term trends due to failure of field crews to consider "almost" dead trees as countable live trees. Hence, it is recommended that another mortality study be conducted that does not use the first measurement after plot establishment.

8.3.6 Calibration Methods

To this author's knowledge, there has been no substantive research efforts designed to determine optimal methods of calibrating growth models for different forecasting objectives. It is an obvious area for future study.

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Appendix I

PRELIMINARY DATA ADJUSTMENTS

Development of the model system requires a sample plot set with total heights and crown ratios available for each tree on a plot. In addition to these variables, trees that are potential data points are required to have increment measurements on DBH, total height, or height to the crown base. With the exception of height to the crown base, these growth measurements were acquired by differencing repeated measurements on permanent plots or (for DBH increment only) directly from increment cores after adjusting for short term changes in bark thickness

Use of the canopy cover vector to develop density measures required that (at least) estimates of crown length and total height be available for each tree on the sample plot at the initial measurement. To accomplish this, several local height-DBH regression equations were developed for each species on each plot. For each plot, each local model was plotted against the actual data with the aid of an interactive program on a computer terminal. One equation was subsequently selected on a visual basis with the primary emphasis being on reasonableness of predictions throughout the range of diameters on the plot. For species represented by only a few trees on the subject plot, the tree samples were merged with a more abundant species. The same process was repeated with a height to crown base - total height model form and used to estimate crown ratios.

On plots where the DBH increment measurements were made with increment cores, no attempt was made to backdate the stand to reconstruct a plausible initial measurement. Rather, past five year tree basal area growth was assumed to be equal to the next five year increment.

No plots of any kind were used that had been harvested between measurements. No plots were used that had measurement intervals less than three or greater than eight years. Plots that had been measured during the middle of the growing season were adjusted to get a "biological growth interval" on the basis of the Jackson State Forest growth study (Bawcom et. al, 1961). This adjustment was applied to DBH growth only. Most of the plots that required this adjustment were measured during the summer months. Height growth for the year was presumed to be completed by April. There were no sample plots in the data sets that had been measured during the months when annual height growth was presummed to be occuring so no adjustments were necessary. For growth intervals that were not an even multiple of five years, the interval growth measurements were linearly adjusted to give an even five year growth measurement.

All of the direct measurements on crown recession are coarse and limited to the Jackson State CFI plot set. Briefly, this data set is composed of approximately 140 plots that have been measured every five years since their establishment in 1958-1960. At the initial measurement, approximately half of the trees on each plot were measured for total height. At every measurement, a vigor code (based partially on crown ratio) was assigned to almost every tree on each plot. The vigor code is for a range in crown ratios of about 10 to 20 percent. On the last two remeasurements, some of these plots were subsampled for total height and had either height to the crown base or actual crown ratio's measured. Based on the last two measurements, it was found that the correlations between actual crown ratios and the midpoint crown ratio of each vigor class was quite satisfactory. Subsequently, on only those plots that had been subsampled for heights and crown ratio on the last two remeasurements, all vigor codes for all five remeasurements on each tree were converted to crown ratio estimates. To each tree sampled, the actual crown ratio measurements were also added and a linear regression of crown ratio on calendar year was estimated. On trees with only two height measurements, height growth was assumed to be linear over the total twenty year time interval. For trees with three height measurements, a linear regression of height on calendar year was estimated. Using both of the estimators, crown recession trends were developed for sample trees.

Appendix II

AGGREGATION RANGES

The following tables show the ranges used in aggregating data into cell classes for different growth models for each species.

Diameter Growth

Variable	Crown Class	Total Height	Crown Ratio	CC_{66}	Site Index
Units	——	Feet	Percent	Percent	Feet
					Redwood
Ranges	Dominant	10- 25	0- 10	0- 33	75-100
	Codminants	25- 50	10- 20	33- 66	100-120
	Intermediates	50- 75	20- 35	66-100	120-145
					Douglas fir
	Suppressed	75-100	35- 50	100-150	90-120
		100-125	50- 70	150-200	120-140
		125-150	70-100	200-300	140-165
					Tanoak
		150-175		300-400	45- 60
		175-200			60- 80
		200-225			80- 95

Height Growth

Variable	DHG5	Crown Ratio	CC_{66}	Crown Class
Units	Feet	Percent	Percent	----
Ranges	0 - 2.5	0- 10	0- 33	Dominant
	2.5- 5.0	10- 20	33- 66	Codominant
	5.0- 7.5	20- 30	66-100	Intermediate
	7.5-10.0	30- 40	100-150	Suppressed
	10.0-12.5	40-100	150-200	
	12.5-15.0		200-300	
	15.0-20.0		300-400	

Crown Recession

Variable	Predicted Height Growth	Crown Length	Crown Ratio	CC_{htcb}	Crown Class
Units	Feet	Feet	Percent	Percent	——
Ranges	0 - 2.5	0- 10	0- 10	0- 50	Dominant
	2.5- 5.0	10- 20	10- 20	50-100	Codominant
	5.0- 7.5	20- 35	20- 30	100-150	Intermediate
	7.5-10.0	35- 50	30- 40	150-200	Suppressed
	10.0-12.5	50- 75	40- 60	200-300	
	12.5-15.0	75-100	60- 80	300-400	
	15.0-20.0	100-125	80-100		
		125-150			

Appendix III

COMPARISONS OF ACTUAL VERSUS SIMULATED PLOT GROWTH

Plot by plot comparisons are tabulated for actual versus simulated basal area growth by calendar period and species for the Union Lumber Company and Jackson State Forest CFI plot data sets. In the tables, 'A' is the actual net basal area growth in square feet per acre for the indicated calendar period. 'P' is the ratio of actual to simulated growth. Summaries of this information are provided in tables 15 and 17 in Chapter 6.

UNION LUMBER COMPANY PLOTS

Redwood

Plot	1942-1951 (A)	1942-1951 (P)	1952-1976 (A)	1952-1976 (P)
p01	8.6	0.89	13.1	0.71
p02	36.0	1.29	96.9	1.17
p04	77.7	1.24	193.0	1.43
p06	29.0	0.98	44.3	0.80
p07	23.5	1.27	27.2	0.70
p08	57.9	1.07	122.8	1.05
p10	35.9	0.99	34.5	0.53
p12	65.3	1.21	159.6	1.43
p13	23.0	1.13	58.1	1.18
p18	33.4	0.89	52.8	0.77
p19	20.3	0.81	42.5	1.07
p20	14.2	0.62	22.1	0.57

Douglas fir

Plot	1942-1951 (A)	1942-1951 (P)	1952-1976 (A)	1952-1976 (P)
p01	35.9	0.76	83.3	0.84
p04	8.5	1.32	32.9	1.78
p05	39.2	0.81	45.1	0.48
p06	48.0	1.33	99.6	1.19
p07	31.6	0.91	100.5	1.16
p08	1.9	0.90	4.0	0.66
p10	33.3	1.17	95.7	1.27
p12	3.3	1.00	10.0	1.20
p13	19.8	1.50	69.0	1.89
p18	27.9	1.09	81.0	1.22
p19	26.9	0.69	92.7	1.01
p20	27.5	0.76	84.0	1.11

JACKSON STATE FOREST CFI PLOTS

Redwood

Plot	1959-1964 (A)	1959-1964 (P)	1964-1969 (A)	1964-1969 (P)	1969-1974 (A)	1969-1974 (P)	1974-1979 (A)	1974-1979 (P)
1010	9.1	1.14	8.1	1.07	8.0	1.10	9.0	1.29
1028	9.4	1.07	8.8	1.05	6.1	0.75	12.2	1.58
1030	23.5	1.27	16.8	0.94	15.2	0.88	20.1	1.21
1034	24.4	1.14	20.5	0.94	18.8	0.87	17.4	0.82
1039	13.2	1.02	14.8	1.25	9.2	0.84	12.7	1.25
1041	21.2	0.75	21.1	0.74	10.0	0.36	13.4	0.49
1044	32.0	1.09	28.1	0.95	20.5	0.70	22.7	0.79
1050	7.8	0.86	7.8	0.93	7.6	0.98	5.3	0.74
1056	21.0	1.00	20.9	0.97	16.4	0.75	18.8	0.86
1057	11.0	0.98	11.2	1.06	9.2	0.93	7.7	0.82
1060	11.7	1.40	8.8	1.02	8.7	0.99	7.2	0.82
1067	13.7	1.70	12.3	1.46	8.5	0.98	2.5	0.28
1070	16.7	1.20	21.2	1.44	13.0	0.86	16.8	1.08
1071	49.5	1.20	37.4	0.94	28.8	0.76	32.0	0.87
1080	13.9	0.83	11.3	0.65	10.4	0.58	9.3	0.52
1083	23.1	0.87	22.4	0.80	14.9	0.52	19.3	0.68
1095	18.3	0.83	19.7	0.84	13.4	0.54	14.4	0.58
1096	21.3	0.76	21.8	0.82	12.9	0.51	17.4	0.74
1022	27.9	0.87	27.8	0.88	22.7	0.74	21.1	0.71
1031	23.8	1.14	23.5	1.19	15.7	0.83	18.5	1.02
1038	15.5	1.08	16.9	1.21	13.3	0.98	15.1	1.15
1043	10.8	0.56	9.0	0.45	9.4	0.47	7.3	0.39
1051	36.7	0.99	34.7	1.00	30.5	0.92	26.3	0.83
1055	19.4	1.35	19.1	1.25	14.7	0.93	18.5	1.14
1061	12.8	0.71	9.8	0.54	6.2	0.34	8.3	0.45
1062	26.5	1.34	16.2	0.85	15.5	0.85	13.1	0.76
1064	14.3	0.48	21.6	0.70	12.3	0.39	19.7	0.61
1068	33.3	1.61	26.9	1.28	15.8	0.75	24.0	1.15
1069	16.3	1.14	4.2	0.28	15.5	0.98	14.9	0.91
1073	24.5	1.31	20.3	1.02	6.6	0.32	18.0	0.84
1074	17.9	0.68	18.9	0.71	6.6	0.25	19.8	0.76
1075	31.7	1.31	18.9	0.80	20.2	0.87	17.1	0.76
1076	24.4	0.89	25.4	0.95	15.2	0.58	23.4	0.92
1081	15.6	0.90	12.9	0.74	10.1	0.59	12.2	0.73
1082	26.7	1.33	17.6	0.86	12.2	0.60	13.6	0.67
1084	30.1	1.54	28.3	1.44	22.4	1.15	31.6	1.65
1097	17.2	1.13	11.3	0.87	15.2	1.37	14.9	1.52
1111	15.0	0.67	16.5	0.73	14.8	0.65	13.7	0.60

Douglas fir

Plot	1959-1964		1964-1969		1969-1974		1974-1979	
	(A)	(P)	(A)	(P)	(A)	(P)	(A)	(P)
1010	10.3	0.83	2.4	0.20	10.3	0.89	6.8	0.61
1019	7.5	1.56	8.0	1.61	6.6	1.31	8.1	1.59
1030	9.8	1.15	8.4	0.99	8.7	1.05	11.2	1.37
1039	20.8	1.17	13.2	0.76	15.2	0.90	15.7	0.96
1050	25.6	1.34	19.4	1.01	23.9	1.29	17.2	0.96
1057	15.9	1.23	16.0	1.15	16.5	1.16	13.4	0.94
1060	9.0	1.33	6.6	0.92	2.5	0.34	8.1	1.08
1072	9.5	2.51	10.2	2.59	10.7	2.67	10.1	2.52
1088	8.4	2.09	7.3	1.70	8.1	1.81	5.5	1.20
1096	13.0	2.02	12.1	1.81	10.2	1.47	11.7	1.65
1022	7.3	1.11	7.0	0.96	7.6	0.99	8.3	1.05
1031	5.2	1.16	4.3	0.93	4.2	0.89	4.4	0.93
1038	3.6	0.90	4.1	0.94	3.5	0.76	5.0	1.05
1043	16.1	1.53	15.4	1.40	15.0	1.34	8.7	0.77
1062	7.2	0.81	6.3	0.65	4.9	0.49	4.8	0.47
1065	22.3	1.29	17.8	0.95	18.2	0.94	11.8	0.61
1074	3.0	1.35	3.1	1.31	3.4	1.39	4.5	1.81
1081	8.2	0.92	9.4	0.99	9.0	0.91	9.5	0.95
1087	9.3	2.07	8.0	1.68	3.3	0.67	8.0	1.59
1097	23.3	1.06	23.5	1.10	19.5	0.93	17.3	0.84